Optimal hierarchical pricing schemes for wireless network usage and resource allocation

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***Abstract*—Typically the cost of a product has many components. Various components correspond to the production chain steps through which the product goes before meeting a customer. This also takes place in the price formation in wireless networks. For instance, before transmitting customer data, a network operator has to buy some frequency range and also establish contracts with electricity providers. In this paper we try to establish the tariff formation scheme in wireless networks. We consider an hierarchical game with three levels: the user, the provider and the authority. The user intends to transmit data on a network. The amount of traffic sent by the user depends on the available frequency bandwidth as well as on the tariff. The amount of frequency bandwidth is negotiated between the provider and the authority. A natural question arises for the provider: which tariff the provider has to assign to get the maximal pure profit, i.e. different between how much he obtains from the user and how much he has to pay for the reserved frequency bandwidth to the authority. The authority also looks for the frequency bandwidth tariff which can bring a maximal profit for him. We consider a Stackelberg game model with three levels of hierarchy: the authority as the leader of the first level, the provider who is the follower for the authority and the leader for the lower level, and the user who is the follower for the provider. The formulas for optimal tariffs at each level are established and some very interesting properties of the equilibrium are investigated. The authority obtains more profit by reducing the bandwidth frequency tariff, meanwhile the provider achieves better profit by increasing the user's rate tariff. In fact, our mathematical model can confirm the opinion that the telecom companies have payed too much for 3G licences. Finally, we note that the main novelty in this paper compared to the standard Stackelberg pricing games extensively investigated in the literature is that we consider the three level hierarchical structure user-provider-authority.**

I. INTRODUCTION

Pricing schemes have been widely studied in networking to control the usage of sparse resource as the frequency bandwidth. In the present work we investigate the following question: What should be a composition of the tariffs in wireless networks. Often the organization of wireless networks is based on the interaction among several economic entities. For instance, a user pays some tariff for a certain data rate. In turn, the service provider pays some costs associated with the wireless network usage such as electricity cost, office renting cost, frequency license, etc. To understand tariff formation process, we propose to deal with the problem as an hierarchical Stackelberg game among various agents of the telecommunication market. Specifically, we consider the following players; the user, the service provider and the authority. These agents operate at three levels. The authority is the owner of the network medium and is considered as the top leader in the hierarchy. The provider rents a quantity of the network medium in order to sell network services to the user. The Stackelberg game approach means that the decision of the authority implies an optimal decision of the provider, which in turns implies an optimal decision for the user. As the first step in this research direction, in this work we consider the framework with only one agent per level of the hierarchy. The case with several users and providers which are competing in their hierarchy levels is left for the future works.

Thus, we focus on a wireless market in which the authority sells frequency bandwidth. This situation is typical in economic models of Cognitive Radio Networks [12]. At the lower level, the provider sells the access bandwidth to the user. The user has his/her utility function and pays according to the transmission rate tariff. A natural question arises for the provider: which tariff the provider has to assign to obtain the maximal pure profit, i.e. the difference between how much he obtains from the user and how much he has to pay for the licensed frequency bandwidth to the authority. The authority in turn looks for the frequency bandwidth tariff which can bring to him maximal profit.

We would like to note that Stackelberg game approach is very popular among researchers dealing with pricing in networks (see, for example, [1], [2], [3], [4], [7], [8]). The main different of the scenario suggested into the present paper from the mentioned above that we deal with three levels hierarchy user-provider-authority and, moreover, the user's payment to the provider is based on the throughput tariff (not power cost). This way the scenario becomes more realistic from the economics point of view.

A. Model description

We introduce the following three level hierarchical Stackelberg game with the players: user, provider and authority, where the players act one by one in three steps:

At the first step, the tariff C_P per throughput unit and the frequency bandwidth W are fixed. It is natural to consider that a strategy of the user is the transmitted power $T \in [0, \bar{T}]$ with

$\bar{T}$ is the maximal power the user can apply. For the user's net utility v_U is defined as the difference between his/her rate utility U and his/her payment to the service provider:

$$v_U(T) = U\left(R(T)\right) - C_P R(T), \tag{1}$$

where $R = R(T,W)$ is the user's throughput which can be expressed by the Shannon capacity:

$$R(T,W) = W \ln\left(1 + \frac{hT}{WN_0 + I}\right), \tag{2}$$

where

- N_0 is the background noise,
- I is the induced noise,
- h is the fading channel gain.

Note that by normalization without loss of generality we can assume that $h = 1$.

The ratio $\frac{hT}{WN_0 + I}$ represents the SINR of the user. As the rate user's utility we consider a logarithmic one (which is a concave function)

$$U(x) = \ln(1 + \gamma x) \text{ with } \gamma > 0 \tag{3}$$

and α-fairness utility (also concave function)

$$U(x) = \frac{1}{1-\alpha} x^{1-\alpha} \text{ with } 0 < \alpha < 1. \tag{4}$$

Note that the logarithmic utility has been used in a very large spectrum of game-theoretical economic problems [9], [10], [11].

At the second step, of the Stackelberg game, the service provider looks for the optimal tariff C_P and which frequency bandwidth W it would like to license from the authority. The payoff to the provider v_P is the difference between how much he earns selling service to the user and how much he has to pay for the licensed frequency bandwidth. Then, the provider's payoff is given as follows:

$$v_P(C_P, W) = C_P R(T) - C_W W, \tag{5}$$

where C_W is the tariff on frequency assigned by the authority at the third step.

At the third step, of the Stackelberg game the authority looks for the optimal tariff C_W it has to assign to get the maximal profit. Thus, the authority gain v_A is given by

$$v_A(C_W) = C_W W. \tag{6}$$

Then we summarize the three-level optimization problem as follows from the authority to the end user.

- The authority maximizes his revenue depending on the tariff C_W per unit of frequency bandwidth assigned to the provider, i.e.
$$\max_{C_W} C_W W.$$
- The provider decides on the quantity of bandwidth W to license from the authority and which per rate tariff C_P the end user has to pay for using it, i.e.
$$\max_{W, C_P} C_P R(T) - C_W W.$$
- Finally, the end user determines his transmitted power T in order to optimize his net utility which is the difference between the rate utility and the price imposed by the provider, i.e.
$$\max_T U\left(R(T,W)\right) - C_P R(T,W),$$
where $R(T,W)$ is the user throughput.

As usual in the hierarchical optimization problems, in order to compute a solution, we consider the optimization problem starting from the bottom optimization level (the end user optimization) to the top level optimization problem (the authority).

II. The Solution of the Game in the General Case

In this section we obtain a solution of the Stackelberg game in the most general setting. To find the optimal user strategy we note that

$$\frac{dv_U}{dT}(T) = \left(U'\left(R(T,W)\right) - C_P\right) R'(T,W).$$

Thus, the optimal user strategy is given as follows:

$$T^*(C_P, W) = \min\left\{\left[R_T^{-1}\left((U')^{(-1)}\left(C_P\right), W\right)\right]_+, \overline{T}\right\}$$

where R_T^{-1} is the inverse function to R with respect to the argument T and $[x]_+ = \max(0,x)$. Then we can present $T^*(C_P, W)$ in a bit more detailed form as follows:

$$T^*(C_P, W) = \begin{cases} 0, & C_P \in I_0, \\ R_T^{-1}\left((U')^{(-1)}\left(C_P\right), W\right), & C_P \in I_1, \\ \overline{T}, & C_P \in I_2, \end{cases} \tag{7}$$

where

$$I_0 = [U'(0), \infty),$$
$$I_1 = (U'(R(\overline{T}, W), U'(0)),$$
$$I_2 = [0, U'(R(\overline{T}, W))].$$

Now we have to find the optimal tariff C_P^* from the provider point of view for a given W. The payoff to the provider is his profit $v_P(C_P, W) = C_P R(T(C_P), W) - C_W W$. Then, by (7), we have

$$v_P(C_P, W) = \begin{cases} -C_W W, & C_P \in I_0, \\ C_P (U')^{(-1)}\left(C_P\right) - C_W W, & C_P \in I_1, \\ C_P \overline{T} - C_W W, & C_P \in I_2. \end{cases} \tag{8}$$

It is clear that $v_P(C_P, W)$ is a negative constant on I_0 and monotonically increasing on I_2. Then, if the inverse rate function is decreasing on I_1, we have the following result.

Theorem 1: Let the function $C_P (U')^{(-1)}\left(C_P\right)$ be decreasing with C_P on the interval $I_1 = (U'(R(\overline{T}, W)),\ U'(0))$. Then the optimal provider tariff is given as follows:

$$C_P^*(W) = U'(R(\overline{T}, W))$$

which guarantees that the user consumes all the frequency bandwidth. Then, the three levels optimization problem can be reduced to sequential solution of the following two maximization problems:

(a) for a fixed C_W, the provider maximizes his revenue depending on the quantity of bandwidth W :

$$W(C_W) = \max_W v_P(W)$$

with

$$v_P(W) := v_P(C_P^*(W), W) = U'(R(\overline{T}, W))R(\overline{T}, W) - C_W W.$$

(b) $\max_{C_W} W(C_W)C_W$,

Note that

$$\frac{dv_P}{dW}(W) = F(W) - C_W, \tag{9}$$

where

$$F(W) = \overline{T}\frac{d}{dW}U'(R(\overline{T}, W)).$$

Assume that $F(W)$ is a decreasing non-negative function on W. Then at the second step the optimal W^* is given as a root of the equation $F(W) = C_W$ and our problem finally is reduced to the following maximizing problem:

$$\text{maximize}_{C_W \in [0, F(0)]} v_A(C_W)$$

with

$$v_A(C_W) = W^*(C_W)C_W, \tag{10}$$

where $W^*(C_W)$ is the root of the equation

$$F(W) = C_W. \tag{11}$$

Note that such C_W has to satisfy the following condition:

$$\frac{dv_A}{dC_W}(C_W) = 0. \tag{12}$$

Also, by (10), we have

$$\frac{dv_A}{dC_W}(C_W) = \frac{dW^*}{dC_W}(C_W)C_W + W^*(C_W)$$

and by (11)

$$\frac{dF}{dW}(W(C_W))\frac{dW}{dC_W}(C_W) = 1.$$

Substituting the last two relations into (12) implies the following result on the optimal bandwidth tariff for the authority.

Theorem 2: Let the two conditions hold:

(i) $C_P(U')^{(-1)}(C_P)$ be decreasing with C_P on the interval $(U'(R(\overline{T}, W)),\ U'(0))$, and

(ii) $F(W)$ be decreasing non-negative function with respect to W.

Then the optimal bandwidth tariff C_W^* for the authority is given as a solution of the equation:

$$C_W + F^{-1}(C_W)\frac{dF}{dW}(F^{-1}(C_W)) = 0.$$

The provider intends to buy $W^* = F^{-1}(C_W^*)$ bandwidth and to assign the following rate tariff to the user

$$C_P^* = U'(R(\overline{T}, W^*)),$$

which allows to the user to employ all the network facilities in full value.

III. Particular cases of the user utility

In this section, we determine explicitly the solutions with particular rate user's utility functions. First, if the user's utility is the logarithmic one given by equation, (3), the conditions of Theorems 1 and 2 hold and we obtain the optimal tariffs. It is easy to see that

$$\frac{dv_U}{dT}(T) = \frac{\gamma - C_P \Xi(T, W)}{(WN_0 + I + T)\Xi(T, W)},$$

where

$$\Xi(T, W) = 1 + \gamma \ln\left(1 + \frac{T}{WN_0 + I}\right).$$

Thus, the optimal user strategy is given as follows:

$$T^*(C_P, W) = \begin{cases} \bar{T}, & C_P \le \dfrac{\gamma}{\Xi(\bar{T}, W)}, \\ (WN_0 + I) & \\ \times\left(\exp\left(\dfrac{\gamma - C_P}{\gamma C_P W}\right) - 1\right), & \dfrac{\gamma}{\Xi(\bar{T}, W)} < C_P < \gamma, \\ 0 & \text{if } \gamma \le C_P. \end{cases}$$

So, the provider's payoff, at the second step of the game, is given as follows:

$$v_P(C_P, W) = \begin{cases} W\sigma(\bar{T}, W)C_P - C_W W, & C_P \le \dfrac{\gamma}{\Xi(\bar{T}, W)}, \\ \dfrac{\gamma - C_P}{\gamma} - C_W W, & \dfrac{\gamma}{\Xi(\bar{T}, W)} < C_P < \gamma, \\ -C_W W, & \gamma \le C_P. \end{cases}$$

where

$$\sigma(T, W) = \ln\left(1 + \frac{T}{WN_0 + I}\right).$$

It is clear that $v_P(C_P, W)$ is decreasing with C_P on the interval $(\frac{\gamma}{\Xi(\bar{T}, W)}, \gamma)$. Thus, the condition of Theorem 1 holds and the optimal provider tariff is given as follows:

$$C_P^* = C_P^*(W) := \frac{\gamma}{\Xi(\bar{T}, W)}.$$

At the second step of the game, the provider's payoff turns into the following form:

$$v_P(W) = 1 - \frac{1}{\Xi(\bar{T}, W)} - C_W W. \tag{13}$$

Then the function $F(W)$ defined in (9), has the form:

$$F(W) := \frac{\gamma}{\Xi(\bar{T}, W)^2} \times \left(\sigma(\bar{T}, W) - \frac{WN_0\bar{T}}{(WN_0 + I + \bar{T})(WN_0 + I)}\right).$$

Now we will show that $F(W)$ is a positive decreasing function in W. To do so we find its derivative:

$$\frac{dF}{dW}(W) = -\gamma \frac{A\sigma^2(\bar{T}, W) + B\sigma(\bar{T}, W) + C}{(WN_0 + I + \bar{T})^2 (WN_0 + I)^2 \Xi(\bar{T}, W)^3},$$

where

$$\begin{aligned} A :=& 2\gamma(WN_0 + I + \bar{T})^2 (WN_0 + I)^2, \\ B :=& -\gamma W\bar{T}N_0(2I^2 + 4W^2N_0^2 + 6IWN_0 + 2I\bar{T} + 3WN_0\bar{T}), \\ C :=& \bar{T}N_0(2\gamma N_0\bar{T}W^2 + N_0\bar{T}W + 2IWN_0 + 2I\bar{T} + 2I^2). \end{aligned}$$

Since $A > 0$ the function $A\xi^2 + B\xi + C$ achieves its minimum at $\xi = -B/(2A)$ and there it is equal to $C - B^2/(4A)$. Direct calculations show that

$$\begin{aligned} C - \frac{B^2}{4A} =& \frac{\bar{T}N_0(2I^2 + 2I\bar{T} + 2IN_0W + N_0W\bar{T})}{8(N_0W + I)^2(N_0W + I + \bar{T})^2} \\ &\times \left[8N_0^3(N_0 + \gamma\bar{T})W^4 \right. \\ &+ N_0^2(16N_0\bar{T} + 32N_0I + 7\bar{T}^2 + 14\gamma\bar{T}I)W^3 \\ &+ 2N_0(4\bar{T}N_0 + 24I\bar{T}N_0 + 24N_0I^2 + 3\gamma\bar{T}I(\bar{T} + I))W^2 \\ &\left. + 16N_0I(\bar{T} + I)(\bar{T} + 2I)W + 8I^2(\bar{T} + I)^2\right] \end{aligned}$$

Thus $\frac{dF}{dW}(W) < 0$ and the conditions of Theorem 1 holds. Hence, we have the following result:

Theorem 3: Let the user rate utility be the logarithmical utility function $U(x) = \ln(1 + \gamma x)$.
The optimal bandwidth tariff C_W^* for the authority is given by the unique solution of the equation:

$$C_W + F^{-1}(C_W)\frac{dF}{dW}(F^{-1}(C_W)) = 0.$$

The provider intends to buy $W^* = F^{-1}(C_W^*)$ bandwidth and to assign the rate tariff to the user

$$C_P^* = \frac{\gamma}{1 + \gamma F^{-1}(C_W^*) \ln\left(1 + \dfrac{\bar{T}}{F^{-1}(C_W^*)N_0 + I}\right)},$$

which allows the user to employ all the network facilities in full.
Also, if $C_W^* = 0$ then $v_P(W)$ is an increasing function such that

$$\text{limit}_{\ W \to \infty} v_P(W) = \frac{\gamma\bar{T}}{N_0 + \gamma\bar{T}}.$$

So, free bandwidth does not cause unlimited increase of the provider's profit.

Finally, note that similar calculation allows us to show that if the user employs α-fairness utility $U(x) = x^{1-\alpha}/(1-\alpha)$ with $\alpha \in (0, 1)$ the result still holds. In this case the rate tariff for the user is given as follows:

$$C_P(W) = \frac{1}{W^\alpha \ln^\alpha\left(\dfrac{WN_0 + I + \bar{T}}{WN_0 + I}\right)},$$

which allows the user to employ all the network facilities in full and

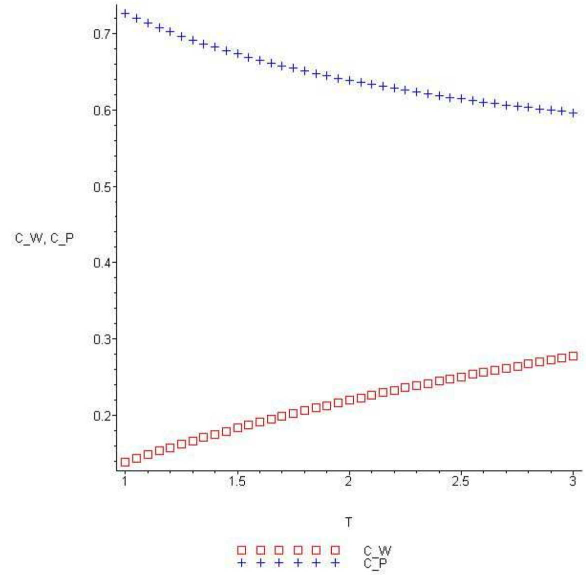

Fig. 1. The optimal tariffs for $\bar{T} \in [1, 3]$

$$\begin{aligned} F(W) :=& \frac{1-\alpha}{\sigma(W,T)^{-\alpha}} \\ &\times \left(\sigma(W,\bar{T}) - \frac{W\bar{T}N_0}{(WN_0 + \bar{T})(WN_0 + \bar{T} + I)}\right). \end{aligned}$$

IV. How players obtain their profit

To come to some conclusion about the optimal players' behaviour, let us consider a numerical example with the background noise $N_0 = 1$, the induced noise $I = 1$, the logarithmic utility function coefficient $\gamma = 1$, and the maximal user power capacity $\bar{T} \in [1, 3]$. Figures 1 and 2 demonstrate how tariffs and the players' profit vary with increasing user's demand. Of course, the provider and authority profits are increasing with increasing user's power capability. This is quite natural. Quite astonishing is the way how they achieve this increase in profit. The authority obtains it by reducing the bandwidth frequency tariff, meanwhile the provider achieves it by increasing the user's rate tariff. All together they act as the unique team taking care about the user in a way that he/she spends the maximal amount of money he/she intends to spent for using network services.

V. Discussion on the low SINR regime

The main difference between the plots suggested into the last section and the ones obtained in Stackelberg pricing games extensively investigated by other researchers, is that we deal with the three level hierarchical structure. This structure is composed of the three players: user, provider and authority. The user's payment to the provider performs according to the throughput tariff (not power cost) which makes the model more realistic. We have not managed to obtain expressions for the optimal tariffs in the closed form. However, for the regime of low SINR we can obtain the expressions for the tariffs in

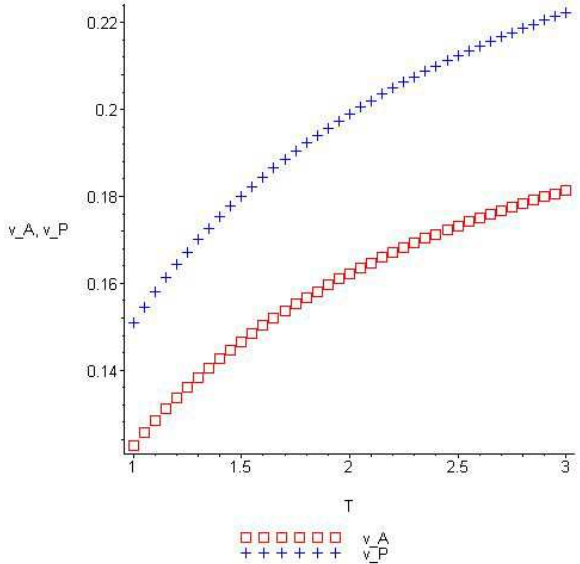

Fig. 2. The optimal profits for $\bar{T} \in [1, 3]$

the closed form. Indeed, for the regime of low SINR, the user throughput can be approximated by the SINR itself, i.e.

$$W \ln\left(1 + \frac{T}{WN_0 + I}\right) \cong W \frac{T}{WN_0 + I}.$$

Thus, the payoff of the user described by equation (1), is given as follows:

$$v_U(T) = \ln\left(1 + \gamma \frac{WT}{WN_0 + I}\right) - \frac{WT}{WN_0 + I} C_P.$$

First we have to find the optimal user strategy. To do this we note that

$$\frac{dv_U}{dT}(T) = W \frac{\gamma(WN_0 + I) - C_P(WN_0 + I + \gamma WT)}{(WN_0 + I)(WN_0 + I + \gamma WT)}.$$

Thus, the optimal user strategy is given as follows

$$T(C_P) = \begin{cases} \bar{T}, & C_P \le \theta(\bar{T}, W)\gamma, \\ \dfrac{(\gamma - C_P)(WN_0 + I)}{\gamma C_P W}, & \theta(\bar{T}, W)\gamma < C_P < \gamma, \\ 0, & \gamma \le C_P. \end{cases}$$

where

$$\theta(T, W) = \frac{WN_0 + I}{WN_0 + I + \gamma WT}.$$

So, the provider's payoff at the second step of the game is given by:

$$\begin{aligned} v_P(C_P, W) &= C_P \frac{WT(C_P)}{WN_0 + I} - C_W W \\ &= \begin{cases} \dfrac{W\bar{T}}{WN_0 + I} C_P - C_W W, & C_P \le \theta(\bar{T}, W)\gamma, \\ \dfrac{\gamma - C_P}{\gamma} - C_W W, & \gamma\theta(\bar{T}, W) < C_P < \gamma, \\ -C_W W, & \gamma \le C_P. \end{cases} \end{aligned}$$

Then, the provider revenue achieves its maximum when the tariff for the user C_P is such that

$$C_P^* = C_P^*(W) := \theta(\bar{T}, W)\gamma.$$

Then, the provider's payoff turns into the following form:

$$v_P(W) := v_P(C_P^*(W), W) = \frac{\gamma W\bar{T}}{WN_0 + I + \gamma W\bar{T}} - C_W W. \tag{14}$$

Note that the derivative of the provider's revenue is:

$$\begin{aligned} \frac{dv_P}{dW}(W) = &\frac{I(\gamma\bar{T} - C_W I) - 2WI(\gamma\bar{T} + N_0)C_W}{(WN_0 + I + \gamma W\bar{T})^2} \\ &- \frac{W^2(\gamma\bar{T} + N_0)^2 C_W}{(WN_0 + I + \gamma W\bar{T})^2}. \end{aligned}$$

Since the quadratical equation

$$I(\gamma\bar{T} - C_W I) - 2WI(\gamma\bar{T} + N_0)C_W - W^2(\gamma\bar{T} + N_0)^2 C_W = 0$$

has two roots

$$W = -\sqrt{\frac{I}{C_W}} \frac{\sqrt{C_W I} \pm \sqrt{\gamma\bar{T}}}{N_0 + \gamma\bar{T}}.$$

we have the following cases:

(a) if

$$C_W > \gamma\bar{T}/I$$

then $v_P(W)$ is decreasing for positive W,

(b) if

$$C_W < \gamma\bar{T}/I$$

then $v_P(W)$ is

$$\begin{cases} \text{increasing in} & \left(0, \sqrt{\dfrac{I}{C_W}} \dfrac{-\sqrt{C_W I} + \sqrt{\gamma\bar{T}}}{N_0 + \gamma\bar{T}}\right), \\ \text{decreasing in} & \left(\sqrt{\dfrac{I}{C_W}} \dfrac{-\sqrt{C_W I} + \sqrt{\gamma\bar{T}}}{N_0 + \gamma\bar{T}}, \infty\right), \end{cases}$$

Thus, the optimal bandwidth $W^*(C_W)$ which maximizes the provider's revenue is given as follows:

$$W^*(C_W) = \begin{cases} 0, & \dfrac{\gamma\bar{T}}{I} \le C_W, \\ \dfrac{\sqrt{\dfrac{\gamma\bar{T}I}{C_W}} - I}{N_0 + \gamma\bar{T}} & C_W < \dfrac{\gamma\bar{T}}{I}. \end{cases}$$

Thus, at the third step of the game, the authority's payoff is given by

$$v_A(C_W) = \begin{cases} 0, & \dfrac{\gamma\bar{T}}{I} \le C_W, \\ \dfrac{\sqrt{\gamma\bar{T}IC_W} - IC_W}{N_0 + \gamma\bar{T}} & C_W < \dfrac{\gamma\bar{T}}{I}. \end{cases}$$

Then, the optimal authority's strategy is to assign the frequency bandwidth tariff C_W^* as follows: $C_W^* = \gamma\bar{T}/4I$. Finally, we have proved the following result supplying the hierarchical equilibrium strategies of the authority, provider and user.

Theorem 4: In the three level hierarchical tariff game with the SINR as the user throughput the Stackelberg equilibrium strategy of the authority is to assign the frequency bandwidth tariff C_W^* as follows

$$C_W^* = \frac{\gamma \bar{T}}{4I}.$$

This tariff allows the provider to employ in its optimal behaviour the following spectrum of frequency bandwidth

$$W^* = \frac{I}{N_0 + \gamma \bar{T}}$$

The optimal tariff C_P^* determined by the provider for the user is given as follows:

$$C_P^* = \frac{WN_0 + I}{WN_0 + I + \gamma h W_* \bar{T}} \gamma,$$

and the optimal user strategy is

$$T^* = \bar{T}.$$

Then, we conclude by observing that

- Both strategies for the authority and the provider have to be maximally greedy to bring the maximal profit. The network is maximally loaded so that $T = \bar{T}$.
- The optimal authority and provider profits coincides, namely,

$$v_A^* = v_P^* = \frac{\gamma \bar{T}}{4(N_0 + \gamma \bar{T})}.$$

  Besides the bandwidth frequency tariff is increasing meanwhile the user rate tariff goes down with increasing user capability ($\bar{T}$). So, it looks like there is some kind of inside cooperative stimulus of all the participants of the market (they just split the common profit) who are in charge for network functionality and going to meet all the user's demands.
- The unlimit extensive development of the equipment (so, unlimited increase of $\bar{T}$) cannot lead to unlimited increase in profit of the authority and provider since they are upper bound by 1/4.
- Absolute perfection of the network (if N_0 tends to 0) also cannot lead to unlimited increase of the provider and authority profits due to the same their upper bound.

VI. Conclusions and perspectives

In this paper we have studied a three-levels tariff game among user, provider and authority to clarify the process of tariff formation. The proposed economic model can be applied in several networking contexts like in cognitive radio network or 3G where the authority rents frequencies to providers and users pay for their transmission rates to the providers. This three-levels model gives interesting results on how optimal economic relations are build between the players. In particular, for low SINR regime we showed that the unlimit extensive development of the equipment cannot lead to unlimited increase in the profits of the authority and provider. For the general SINR regime we proved that free bandwidth does not cause unlimited increase in the provider's profit. Also we show that the provider and authority archive increase in their profits in quite surprisingly different ways. The authority achieves his profit by reducing the bandwidth frequency tariff, meanwhile the provider archives his profit by increasing the user's rate tariff.

In perspectives, we would like to introduce some demand function for the user since the usage of the network, and then the revenue of the provider, depend on not only the tariff but also on the quality perceived by the user. Also, we plan to extend the current model to the case of several network users and several providers.

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