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Abstract: We propose a novel job scheduling approach for homogeneous cluster computing platforms. Its key feature is the use of virtual machine technology to share *fractional* node resources in a precise and controlled manner. Other VM-based scheduling approaches have focused primarily on technical issues or on extensions to existing batch scheduling systems, while we take a more aggressive approach and seek to find heuristics that maximize an objective metric correlated with job performance. We derive absolute performance bounds and develop algorithms for the online, non-clairvoyant version of our scheduling problem. We further evaluate these algorithms in simulation against both synthetic and real-world HPC workloads and compare our algorithms to standard batch scheduling approaches. We find that our approach improves over batch scheduling by orders of magnitude in terms of job stretch, while leading to comparable or better resource utilization. Our results demonstrate that virtualization technology coupled with lightweight online scheduling strategies can afford dramatic improvements in performance for executing HPC workloads.

Key-words: cluster, scheduler, virtual machine, vector binpacking, high performance computing, batch scheduling, stretch

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Ordonnancement dynamique et fractionnaire des ressources versus ordonnancement par *batch*

Résumé : Nous proposons une nouvelle approche de l'ordonnancement des applications sur les calculateurs parallèles homogènes. Sa principale caractéristique est l'utilisation de machines virtuelles pour organiser le partage de *fractions* des ressources de manière précise et contrôlée. Les approches existantes utilisant des machines virtuelles se sont principalement intéressées à des problèmes techniques ou à l'extension des systèmes de *batch* existants. Notre approche est beaucoup plus aggressive et nous recherchons des heuristiques qui optimisent une métrique particulière. Nous établissons des bornes de performance absolues et nous développons des algorithmes pour la version en-ligne, non clairvoyante, de notre problème d'ordonnancement. Nous évaluons ces algorithmes au moyen de simulations impliquant soit des traces synthétiques, soit de traces d'un système HPC existant. Nous comparons par ce moyen nos solutions aux algorithmes d'ordonnancement par *batch* les plus classiques. Nous montrons que notre approche permet d'améliorer de plusieurs ordres de grandeur le facteur de ralentissement (*stretch*) subit par les applications par rapport aux systèmes de *batch*, tout en ayant une utilisation comparable ou moindre des ressources. Nos résultats montrent que l'utilisation conjointe des techniques de virtualisation et de stratégies d'ordonnancement en-ligne permet d'améliorer très significativement l'exécution des applications dans les systèmes de calcul HPC.

Mots-clés : ordonnancement, machines virtuelles, *bin packing* vectoriel, calcul haute performance, ordonnancement par *batch*, *stretch*

1 Introduction

The standard method for sharing a cluster among High Performance Computing (HPC) users is batch scheduling. With batch scheduling, users submit *requests* to run applications, or *jobs*. Each request is placed in a queue and waits to be granted an *allocation*, that is, a subset of the cluster's compute nodes, or *nodes* for short. The job has exclusive access to these nodes for a bounded duration.

One problem with batch scheduling is that it inherently limits overall resource utilization. If a job uses only a fraction of a node's resource (e.g., half of the processor cores, a third of the memory), then the remainder of it is wasted. It turns out that this is the case for many jobs in HPC workloads. For example, in a 2006 log of a large Linux cluster [1], more than 95% of the jobs use under 40% of a node's memory, and more than 27% of the jobs effectively use less than 50% of the node's CPU resource. Similar observations have been made repeatedly in the literature [2–5]. Additionally, since batch schedulers use integral resource allocations with no time-sharing of nodes, incoming jobs can be postponed even while some nodes are sitting idle.

A second problem is the known disconnect with user concerns (response time, fairness) [6, 7]. While batch schedulers provide myriad configuration parameters, these parameters are not directly related to relevant user-centric metrics.

In this work we seek to remedy both of the above problems. We address the first by allowing fractional resource allocations (e.g., allocating 70% of a resource to a job task) that can be modified on the fly (e.g., by changing allocated fractions, by migrating job tasks to different nodes). We address the second problem by defining an objective performance metric and developing algorithms that attempt to optimize it.

Existing job scheduling approaches generally assume that job processing times are known [8] or that reliable estimates are available [9]. Unfortunately, user-provided job processing time estimates are often inaccurate [10], albeit used by batch schedulers. We take a drastic departure from the literature and assume no knowledge of job processing times.

Our approach, which we term *dynamic fractional resource scheduling* (DFRS), amounts to a carefully controlled time-sharing scheme enabled by virtual machine (VM) technology. Other VM-based scheduling approaches have been proposed, but the research in this area has focused primarily on technical issues [11,12] or extensions to existing scheduling schemes, such as combining best-effort and reservation based jobs [13]. In this work we:

- Define the offline and online DFRS problems and establish their complexity;
- Derive absolute performance bounds for any given problem instance;
- Propose algorithms for solving the online non-clairvoyant DFRS problem;
- Evaluate our algorithms in simulation using synthetic and real-world HPC workloads;
- Identify algorithms that outperform batch scheduling by orders of magnitude;
- Define a new metric to capture the notion of efficient resource utilization;
- Demonstrate that our algorithms can be easily tuned so that they are as or more resource efficient than batch scheduling while still outperforming it by orders of magnitude.

We formalize the DFRS problem in Section 2, study its complexity in Section 3, and propose DFRS algorithms in Section 4. We describe our experimental methodology in Section 5 and present results in Section 6. We discuss related work in Section 7, and conclude with a summary of results and a highlights of future directions in Section 8.

2 The DFRS Approach

DFRS implements fractional allocation of resources, such as CPU cycles, and thus uses time-sharing. The classical time-sharing solution for parallel applications is gang scheduling [14]. In gang scheduling, tasks in a parallel job are executed during the same synchronized time slices across cluster nodes. Gang scheduling requires distributed synchronized context-switching, which may require significant overhead and thus long time slices. Furthermore, in any time slice the use of any node is dedicated to a single application, which leads to low system utilization for applications that do not saturate CPU resources. Because of its drawbacks, gang scheduling is used far less often than batch scheduling for managing HPC clusters.

To circumvent the problems of batch scheduling without being victim of the drawbacks of gang scheduling, in this work we opt for time-sharing in an uncoordinated and low-overhead manner, enabled by virtual machine (VM) technology. Beyond providing mechanisms for efficient time-sharing, VM technology also allows for seamless job preemption and migration without any modification of application code. Both preemption and migration can be used to increase fairness among jobs and, importantly, to avoid starvation. Migration can also be used to achieve better load balance, and hence better system utilization and better overall job performance.

2.1 System Overview and Use of VM Technology

We target clusters of homogeneous *nodes* managed by a resource allocation system that relies on VM technology. The system responds to job requests by creating collections of VM instances on which to run the jobs. Each VM instance runs on a physical node under the control of a VM Monitor that can enforce specific resource fraction allocations for the instance. All VM Monitors are in turn under the control of a VM Manager that specifies allocated resource fractions for all VM instances. The VM Manager can also preempt instances, and migrate instances among physical nodes. Several groups in academia and industry have developed systems with this conceptual architecture [15–19]. Such use of VM technology as the main resource consolidation and management mechanism is today one of the tenets of "cloud computing."

VM technology allows for accurate sharing of hardware resources among VM instances while achieving performance isolation. For instance, the popular Xen VM Monitor [20] enables CPU-sharing and performance isolation in a way that is low-overhead, accurate, and rapidly adaptable [21]. Furthermore, sharing can be arbitrary. For instance, the Xen Credit CPU scheduler can allow three VM instances to each receive 33.3% of the total CPU resource of a dual-core machine [22]. This allows a multi-core physical node to be considered as an arbitrarily time-shared single core. Virtualization of other resources, such as I/O resources, is more challenging [23] but is an active area of research [24]. Recent work also targets the virtualization of full memory hierarchies (buses and caches) [25]. In this work we simply assume that one can start a VM instance on a node and allocate to it reasonably precise fractions of the resources on that node. Given this capability, whether available today or in the future, our approach is applicable to many resource dimensions. In our experiments we include solely CPU and memory resources, the sharing of which is well supported by VM technology today.

An additional advantage of executing jobs within VM instances is that their instantaneous resource needs can be discovered via monitoring [16, 26], introspection [27, 28], and/or configuration variation [27, 28].

2.2 Problem Statement

We consider a homogeneous cluster based on a switched interconnect and with a network-attached storage. Users submit requests to run jobs that consist of one or more tasks to be executed in parallel. Each task runs within a VM instance. Our goal is to make sound resource allocation decisions. These decisions include selecting initial nodes for VM instances, setting allocated resource fractions for each instance, migrating instances between nodes, preempting and pausing instances (by saving them to local or network-attached storage), and postponing incoming job requests.

Each task has a *memory requirement*, expressed as a fraction of node memory, and a *CPU need*, which is the fraction of node CPU cycles that the task needs to run at maximum speed. For instance, a task could require 40% of the memory of a node and would utilize 60% of the node's CPU resource in dedicated mode. We assume that these quantities are known and do not vary throughout job execution. Memory requirements could be specified by users or be discovered on-the-fly, along with CPU needs, using the discovery techniques described in Section 2.1. Memory capacities of nodes should not be exceeded. In other words, we do not allow the allocation of a node to a set of tasks whose cumulative memory requirement exceeds 100%. This is to avoid the use of process swapping, which can have a hard to predict but almost always dramatic impact on task execution times. We do allow for overloading of CPU resources, meaning that a node may be allocated to a set of tasks whose cumulative CPU needs exceed 100%. Further, the CPU fraction actually allocated to the task can change over time, e.g., it may need to be decreased due to the system becoming more heavily loaded. When a task is given less than its CPU need we assume that its execution time is increased proportionally. The task then completes once the cumulative CPU resource

assigned to it up to the current time is equal to the product of its CPU need and execution time on a dedicated system. Note that a task can never be allocated more CPU than its need. In this work we target HPC workloads, which mostly comprise regular parallel applications. Consequently, we assume that all tasks in a job have the same memory requirements and CPU needs, and that they must progress at the same rate. We enforce that allocations provide identical instantaneous CPU fractions to all tasks of a job, as non-identical fractions needlessly waste resources.

One metric commonly used to evaluate batch schedules is the *stretch* (or slowdown) [29]. The stretch of a job is defined as its actual turn-around time divided by its turn-around time had it been alone on the cluster. For instance, a job that could have run in 2 hours on the dedicated cluster but instead runs in 4 hours due to competition with other jobs experiences a stretch of 2. In the literature a proposed way to optimize both for average performance and for fairness is to minimize the maximum stretch [29], as opposed to simply minimizing average stretch, the latter being prone to starvation [30]. Maximum stretch minimization is known to be theoretically difficult. Even in clairvoyant settings there does not exist any constant-ratio competitive algorithm [30], as seen in Section 3. Nevertheless, heuristics can lead to good results in practice [30].

Stretch minimization, and especially maximum stretch minimization, tends to favor short jobs, but on real clusters the jobs with shortest running times are often those that fail at launch time. To prevent our evaluation of schedule quality from being dominated by these faulty jobs, we adopt a variant of the stretch called the *bounded stretch*, or "bounded slowdown" utilizing the terminology in [31]. In this variant, the turn-around time of a job is replaced by a threshold value if this turn-around time is smaller than that threshold. We set the threshold to 10 seconds, and hereafter we use the term stretch to mean bounded stretch.

In this work we do not assume *any* knowledge about job processing times. Batch scheduling systems require that users provide processing time estimates, but these estimates are typically (wildly) inaccurate [10]. Relying on them is thus a losing proposition. Instead, we define a new metric, the *yield*, that does not use job processing time estimates. The *yield* of a task is the instantaneous fraction of the CPU resource of the node allocated to the task divided by the task's CPU need. Since we assume that all tasks within a job have identical CPU needs and are allocated identical CPU fractions, they all have the same yield which is then the yield of the job. Both the yield and the stretch capture an absolute level of job "happiness", and are thus related. In fact, the yield can be seen as the inverse of an instantaneous stretch. We contend that yield optimization is more feasible than stretch optimization given that job processing times are notorious for being difficult to estimate while CPU needs can be discovered (see Section 2.1). In fact, in our experiments we make the conservative assumption that all jobs are CPU bound (thereby not relying on CPU need discovery).

Our goal is to develop algorithms that explicitly seek to maximize the minimum yield. The key questions are whether this strategy will lead to good stretch values in practice, and whether DFRS will be able to outperform standard batch scheduling algorithms.

3 Theoretical Analysis

In this section we study the offline scenario so that we can derive a lower bound on the optimal maximum stretch of any instance assuming a clairvoyant scenario. We then quantify the difficulty of the online, non-clairvoyant case. Even in an offline scenario and even when ignoring CPU needs, memory constraints make the problem NP-hard in the strong sense since it becomes a bin-packing problem [32]. Consequently, in this section we assume that all jobs have null memory requirements, or, conversely, that memory resources are infinite.

3.1 The Offline Case

Formally, an instance of the offline problem is defined by a set of jobs, $\mathcal{J}$, and a set of nodes, $\mathcal{P}$. Each job j has a set, $\mathcal{T}_j$, of tasks and a CPU need, c_j, between 0 and 1. It is submitted at its *release date* r_j and has *processing time* p_j, representing its execution time on an equivalent dedicated system. A target value $\mathcal{S}$ for the maximum stretch defines a deadline $d_j = r_j + \mathcal{S} \times p_j$ for the execution of each job j. The set of job release dates and deadlines, $\mathbb{D} = \bigcup_{j \in \mathcal{J}} \{r_j, d_j\}$, gives rise naturally to a set $\mathcal{I}$ of consecutive,

non-overlapping, left-closed intervals that cover the time span $[\min_{j\in\mathcal{J}} r_j, \max_{j\in\mathcal{J}} d_j)$, where the upper and lower bounds of each interval in $\mathcal{I}$ are members of $\mathbb{D}$ and each member of $\mathbb{D}$ is either the upper or lower bound of at least one member of $\mathcal{I}$. For any interval t we define $\ell(t) = \sup t - \inf t$ (i.e., $\ell(t)$ is the *length* of t).

A *schedule* is an allocation of processor resources to job tasks over time. For a schedule to be valid it must satisfy the following conditions: 1) every task of every job j must receive $c_j \times p_j$ units of work over the course of the schedule, 2) no task can begin before the release date of its job, 3) at any given moment in time, no more than 100% of any CPU can be allocated to running tasks, 4) the schedule can be broken down into a sequence of consecutive, non-overlapping time spans no larger than some time quantum $\mathcal{Q}$, such that over each time span every task of a job j receives the same amount of work and each of these tasks receives no more than c_j times the length of the time span units of work. Within these small time spans any task can fully utilize a CPU resource, regardless of the CPU need of its job, and the tasks of a job can proceed independently. The exact size of $\mathcal{Q}$ depends upon the system, but as the timescale of parallel job scheduling is orders of magnitude larger than that of local process scheduling, we make the reasonable assumption that for every $t \in \mathcal{I}$, $\mathcal{Q} << \ell(t)$.

Theorem 1. *Let us consider a system with infinite memory and assume that any task can be moved instantaneously and without penalty from one node to another. Then there exists a valid schedule whose maximum stretch is no greater than $\mathcal{S}$ if and only if the following linear system has a solution, where each variable α_j^t represents the portion of job j completed in time interval t:*

$$\begin{cases} \text{(1a)} \quad \forall j \in \mathcal{J} & \sum_{t\in\mathcal{I}} \alpha_j^t = 1; \\ \text{(1b)} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{I} & r_j \geq \sup t \Rightarrow \alpha_j^t = 0; \\ \text{(1c)} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{I} & d_j \leq \inf t \Rightarrow \alpha_j^t = 0; \\ \text{(1d)} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{I} & \alpha_j^t p_j \leq \ell(t); \\ \text{(1e)} \quad \forall t \in \mathcal{I} & \sum_{j\in\mathcal{J}} \alpha_j^t p_j c_j |\mathcal{T}_j| \leq |\mathcal{P}| \ell(t). \end{cases} \tag{1}$$

Proof. The constraints in Linear System (1) are straightforward. They restate a number of the conditions required for a valid schedule in terms of α_j^t, and also add a requirement that jobs be processed before their deadlines. In greater detail:

- Constraint (1a) states that each job must be fully processed;
- Constraint (1b) states that no work can be done on a job before its release date;
- Constraint (1c) states that no work can can be done on a job after its deadline;
- Constraint (1d) states that a task cannot run longer during a time interval than the length of the time interval;
- Constraint (1e) states that the cumulative computational power used by the different tasks during a time interval cannot exceed what is available.

These conditions are necessary. We now show that they suffice to insure the existence of a schedule that achieves the desired maximum stretch (i.e., there exists a valid schedule in which each job completes before its deadline).

From any solution of Linear System (1) we can build a valid schedule. We show how to build the schedule for a single interval t, the whole schedule being obtained by concatenating all interval schedules. For each job j, each of its tasks receives a cumulative computational power equal to $\alpha_j^t p_j c_j$ during interval t. Let a_j^t and b_j^t be two integers such that $\frac{a_j^t}{b_j^t} = \alpha_j^t p_j c_j$. Without loss of generality, we can assume that all integers b_j^t are equal to a constant b: $\forall j \in \mathcal{J}, \forall t \in \mathcal{I}, b_j^t = b$. Let $\mathcal{R}$ be a value smaller than $\mathcal{Q}$ such that there exists an integer $\lambda > 0$ where $\ell(t) = \lambda \times \mathcal{R}$. As $\mathcal{R}$ is smaller than $\mathcal{Q}$, during any of the λ sub-intervals of size $\mathcal{R}$ the different tasks of a job can fully saturate the CPU and be run in any order. During each of these sub-intervals, we greedily schedule the tasks on the nodes in any order: starting at time 0, we first run the first task on the first node at full speed (100% CPU utilization) for a time $\frac{a_{j_1}^t}{b\lambda}$, where j_1 is the job this

task belongs to. Then we run the second task on the first node at full speed (100% CPU utilization) for a time $\frac{a^t_{j_2}}{b\lambda}$, where j_2 is the job this task belongs to. If there is not enough remaining time on the first node to accommodate the second task, we schedule the second task on the first node for all the remaining time, and we schedule the remaining of the task on the second node starting at the beginning of the sub-interval (thanks to our assumption on task migration). We proceed in this manner until every task is scheduled. We now show that this schedule is valid.

Note that our construction ensures that no task runs simultaneously on two different nodes. Indeed, this could only happen if, for a task of some job j, we had:

$$\frac{a^t_j}{\lambda b} > \mathcal{R} \quad \Leftrightarrow \quad \alpha^t_j p_j c_j = \frac{a^t_j}{b} > \ell(t) \ ,$$

which is forbidden by Constraint (1d). Then, by construction, 1) every task of every job j receives $\sum_{t\in\mathcal{I}} \alpha^t_j p_j c_j = p_j c_j$ units of work, 2) no task is started before the release date of its job, 3) at any given time no more than 100% of any CPU is allocated to running tasks, and 4) the schedule can be broken down into a sequence of non-overlapping time spans of size no larger than $\mathcal{Q}$, such that over each time span every task of each job j receives the same amount of work and no task receives more than c_j times the length of the time span units of work. □

Since all variables of Linear System (1) are rational, one can check in polynomial time whether this system has a solution. Using a binary search, one can use the above theorem to find an approximation of the optimal maximum stretch. In fact, one can find the optimal value in polynomial time using a binary search and a version of Linear System (1) tailored to check the existence of a solution for a range of stretch values (see [30, Section 6] for details). While the underlying assumptions in Theorem 1 are not met in practice, the optimal maximum stretch computed via this theorem is a lower bound on the actual optimal maximum stretch.

3.2 The Online Case

Online maximum stretch minimization is known to be theoretically difficult. Recall that the competitive ratio of an online algorithm is the worst-case ratio of the performance of that algorithm with the optimal offline algorithm. Even in a clairvoyant scenario there is no constant-ratio competitive online algorithm [30].

In this work we study an online, non-clairvoyant scenario. However, unlike the work in [30], time-sharing of compute nodes is allowed. The question then is whether this added time-sharing capability can counter-balance the fact that the scheduler does not know the processing time of jobs when they arrive in the system. In general, bounds on the competitive ratios of online algorithms can be expressed as a function of the number of jobs submitted to the system, here denoted by $|\mathcal{J}|$, or as a function of Δ, the ratio between the processing times of the largest and shortest jobs.

In this section we assume that we have one single-core node at our disposal or, equivalently, that all jobs are perfectly parallel. We show that, in spite of this simplification, the problem is still very difficult (i.e., lower bounds on competitive ratios are large). As a result, the addition of time-sharing does not change the overall message of the work in [30]. The first result is that the bound derived for online algorithms in a clairvoyant setting without time-sharing holds in a non-clairvoyant, time-sharing context.

Theorem 2. *There is no $\frac{1}{2}\Delta^{\sqrt{2}-1}$-competitive preemptive time-sharing online algorithm for minimizing the maximum stretch if at least three jobs have distinct processing times.*

This result is valid for both clairvoyant and non-clairvoyant scenarios and is established by the proof of Theorem 14 in [30], which holds when time-sharing is allowed. Surprisingly, we were not able to increase this bound by taking advantage of non-clairvoyance. However, as seen in the next theorem, non-clairvoyance makes it possible to establish a very large bound with respect to the number of jobs.

Theorem 3. *There is no (preemptive) online algorithm for the non-clairvoyant minimization of max-stretch whose competitive ratio is strictly smaller than $|\mathcal{J}|$, the number of jobs.*

Proof. By contradiction, let us hypothesize that there exists an algorithm with a competitive ratio strictly smaller than $|\mathcal{J}| - \varepsilon$ for some $\varepsilon > 0$.

We consider an instance where jobs are all released at time 0, with processing times large enough such that all jobs are kept running until time $|\mathcal{J}|$ regardless of what the algorithm does. The job that has received the smallest cumulative compute time up to time $|\mathcal{J}|$ has received at most 1 unit of compute time (one $|\mathcal{J}|$-th of the $|\mathcal{J}|$ time units). We sort the jobs in increasing order of the cumulative compute time each has received up to time $|\mathcal{J}|$. For some value of $\lambda \geq |\mathcal{J}|$, we construct our instance so that the i-th job in this order has processing time λ^{i-1}. The completion time of job 1, i.e., the job that has received the smallest cumulative compute time up to time $|\mathcal{J}|$, is at least $|\mathcal{J}|$. Consequently, its stretch is no smaller than $|\mathcal{J}|$ because its processing time is $\lambda^0 = 1$.

A possible schedule would have been to execute jobs in order of increasing processing time. The stretch of the job of processing time λ^{i-1} would then be:

$$\frac{\sum_{k=1}^{i} \lambda^{k-1}}{\lambda^{i-1}} = \frac{\lambda^i - 1}{\lambda^{i-1}(\lambda - 1)} \xrightarrow[\lambda \to +\infty]{} 1.$$

Therefore, if λ is large enough, no job has a stretch greater than $1 + \frac{\varepsilon}{|\mathcal{J}|}$ in this schedule. Consequently, the competitive ratio of our hypothetical algorithm is no smaller than:

$$\frac{|\mathcal{J}|}{1 + \frac{\varepsilon}{|\mathcal{J}|}} \geq |\mathcal{J}|(1 - \frac{\varepsilon}{|\mathcal{J}|}) = |\mathcal{J}| - \varepsilon \,,$$

which is a contradiction. □

The EQUIPARTITION algorithm, which gives each job an equal share of the platform, is known to deliver good performance in some non-clairvoyant settings [33]. We therefore assess its performance for maximum stretch minimization.

Theorem 4. *In a non-clairvoyant scenario:*
1) EQUIPARTITION *is exactly a $|\mathcal{J}|$-competitive online algorithm for maximum stretch minimization;*
2) There exists an instance for which EQUIPARTITION *achieves a maximum stretch at least $\frac{\Delta+1}{2+\ln(\Delta)}$ times the optimal.*

Proof.
Competitive ratio as a function of $|\mathcal{J}|$ –
At time t, EQUIPARTITION gives each of the $m(t)$ not-yet-completed jobs a share of the node equal to $\frac{1}{m(t)} \geq \frac{1}{|\mathcal{J}|}$. Hence, no job has a stretch greater than $|\mathcal{J}|$ and the competitive ratio of EQUIPARTITION is no greater than $|\mathcal{J}|$. We conclude using Theorem 3.
Competitive ratio as a function of Δ –
Let us consider a set $\mathcal{J}$ of $n = |\mathcal{J}|$ jobs as follows. Jobs 1 and 2 are released at time 0 and have the same processing time. For $i = 3, \ldots, n$, job j_i is released at time $r_{j_i} = r_{j_{i-1}} + p_{j_{i-1}}$. Job processing times are defined so that, using EQUIPARTITION, all jobs complete at time $r_{j_n} + n$. Therefore, the i-th job is executed during the time interval $[r_{j_i}, r_{j_n} + n]$. There are two active jobs during the time interval $[r_{j_1} = r_{j_2} = 0, r_{j_3}]$, each receiving one half of the node's processing time. For any $i \in [3, n-1]$, there are i active jobs in the time interval $[r_{j_i}, r_{j_{i+1}}]$, each receiving a fraction $1/i$ of the processing time. Finally, there are n jobs active in the time interval $[r_{j_n}, r_{j_n} + n]$, each receiving a fraction $1/n$ of the node's compute time.

The goal of this construction is to have the n-th job experience a stretch of n. However, by contrast with the previous theorem, the value of Δ is "small," leading to a large competitive ratio as a function of Δ, but smaller than n. Formally, to define the job processing times, we write that the processing time of a job is equal to the cumulative compute time it is given between its release date and its deadline using EQUIPARTITION:

$$\begin{cases} \forall i \in [1,2] \quad p_{j_i} = \frac{1}{2}(r_{j_3} - r_{j_1}) + \sum_{k=3}^{n-1} \frac{1}{k}(r_{j_{k+1}} - r_{j_k}) + \frac{1}{n}((r_{j_n} + n) - r_{j_n}) = \frac{1}{2}p_{j_1} + \sum_{k=3}^{n-1} \frac{1}{k} p_{j_k} + 1 \\ \forall i \in [3,n] \quad p_{j_i} = \sum_{k=i}^{n-1} \frac{1}{k}(r_{j_{k+1}} - r_{j_k}) + \frac{1}{n}((r_n + n) - r_n) = \sum_{k=i}^{n-1} \frac{1}{k} p_{j_k} + 1 \end{cases}$$

We first infer from the above system of equations that $p_{j_n} = 1$ (and the n-th job has a stretch of n). Then, considering the equation for p_{j_i} for $i \in [3, n-1]$, we note that $p_{j_i} - p_{j_{i+1}} = \frac{1}{i} p_{j_i}$. Therefore, $p_{j_i} = \frac{i}{i-1} p_{j_{i+1}}$ and, by induction, $p_{j_i} = \frac{n-1}{i-1}$. We also have $p_{j_2} - p_{j_3} = \frac{1}{2} p_{j_1} = \frac{1}{2} p_{j_2}$. Therefore, $p_{j_2} = 2p_{j_3} = n-1$.

Now let us consider the schedule that, for $i \in [2, n]$, executes job j_i in the time interval $[r_{j_i}, r_{j_{i+1}} = r_{j_i} + p_{j_i}]$, and that executes the first job during the time interval $[r_{j_n} + p_{j_n} = r_{j_n} + 1, r_{j_n} + n]$. With this schedule all jobs have a stretch of 1 except for the first job. The maximum stretch for this schedule is thus the stretch of the first job. The makespan of this job, i.e., the time between its release data and its completion, is:

$$\sum_{i=1}^{n} p_{j_i} = 2 \times p_{j_1} + \sum_{i=3}^{n} p_{j_i} = 2(n-1) + \sum_{i=3}^{n} \frac{n-1}{i-1} = (n-1)\left(1 + \sum_{i=2}^{n} \frac{1}{i-1}\right) = (n-1)\left(1 + \sum_{i=1}^{n-1} \frac{1}{i}\right).$$

The first job being of size $n-1$, its stretch is thus: $1 + \sum_{i=1}^{n-1} \frac{1}{i} = 2 + \sum_{i=2}^{n-1} \frac{1}{i}$. Using a classical bounding technique:

$$\sum_{i=2}^{n-1} \frac{1}{i} \leq \sum_{i=2}^{n-1} \int_{i-1}^{i} \frac{1}{x} dx = \int_{1}^{n-1} \frac{1}{x} dx = \ln(n-1).$$

The competitive ratio of EQUIPARTITION on that instance is no smaller than the ratio of the maximum stretch it achieves (n) and of the maximum stretch of any other schedule on that instance. Therefore, the competitive ratio of EQUIPARTITION is no smaller than:

$$\frac{n}{2 + \sum_{i=2}^{n-1} \frac{1}{i}} \geq \frac{n}{2 + \ln(n-1)} = \frac{\Delta + 1}{2 + \ln(\Delta)},$$

as the smallest job —the n-th one— is of size 1, and the largest ones —the first two jobs— are of size $n-1$. □

To put the performance of EQUIPARTITION into perspective, First Come First Served (FCFS) is exactly Δ-competitive [30].

4 DFRS Algorithms

The theoretical results from the previous section indicate that non-clairvoyant maximum stretch optimization is "hopeless": no algorithm can be designed with a low worst-case competitive ratio because of the large number of jobs and/or the large ratio between the largest and smallest jobs found in HPC workloads. Instead, we focus on developing non-guaranteed algorithms (i.e., heuristics) that perform well in practice, hopefully close to the offline bound given in Section 3.1. These algorithms should not use more than a fraction of the bandwidth available on a reasonable high-end system (for task preemption/migration). Additionally, because of the online nature of the problem, schedules should be computed quickly.

We propose to adapt the algorithms we designed in our study of the offline resource allocation problem for static workloads [34, 35]. Due to memory constraints, it may not always be possible to schedule all currently pending jobs simultaneously. In the offline scenario, this leads to a failure condition. In the online scenario, however, some jobs should be run while others are suspended or postponed. It is therefore necessary to establish a measure of priority among jobs. In this section we define and justify our job priority function (Section 4.1), describe the greedy (Section 4.2) and vector packing based (Section 4.3) task placement heuristics, explain how these heuristics can be combined to create heuristics for the online problem (Sections 4.4 and 4.5), give our basic strategies for resource allocation once tasks are mapped to nodes (Section 4.6), and finally provide an alternate algorithm that attempts to optimize stretch directly instead of relying on the yield (Section 4.7).

4.1 Prioritizing Jobs

We define a priority based on the *virtual time* of a job, that is, the total subjective execution time experienced by a job. Formally, this is the integral of the job's yield since its release date. For example, a job that starts

and runs for 10 seconds with a yield of 1.0, that is then paused for 2 minutes, and then restarts and runs for 30 seconds with a yield 0.5 has experienced 25 total seconds of virtual time ($10 \times 1.0 + 120 \times 0.0 + 30 \times 0.5$). An intuitive choice for the priority function is the inverse of the virtual time: the shorter the virtual time, the higher the priority. A job that has not yet been allocated any CPU time has a zero virtual time, i.e., an infinite priority. This ensures that no job is left waiting at its release date, especially short jobs whose stretch would degrade the overall performance. This rationale is similar to that found in [36].

This approach, however, has a prohibitive drawback: The priority of paused jobs remains constant, which can induce starvation. Thus, the priority function should also consider the *flow time* of a job, i.e., the time elapsed since its submission. This would prevent starvation by ensuring that the priority of any paused job increases with time and tends to infinity.

Preliminary experimental results showed that using the inverse of the virtual time as a priority function leads to good performance, while using the ratio of flow time to virtual time leads to poor performance. We believe that this poor performance is due to the priorities of jobs all converging to some constant value related to the average load on the system. As a result, short-running jobs, which suffer a greater penalty to their stretch when paused, are not sufficiently prioritized over longer-running jobs. Consequently, we define the priority function as: priority $= \frac{\text{flow time}}{(\text{virtual time})^2}$. The power of two is used to increase the importance of the virtual time with respect to the flow time, thereby giving an advantage to short-running jobs. We break ties between equal-priority jobs by considering their order of submission.

4.2 Greedy Task Mapping

A basic greedy algorithm, which we simply call Greedy, allocates nodes to an incoming job j without changing the mapping of tasks that may currently be running. It first identifies the nodes that have sufficient available memory to run at least one task of job j. For each of these nodes it computes its *CPU load* as the sum of the CPU needs of all the tasks currently allocated to it. It then assigns one task of job j to the node with the lowest CPU load, thereby picking the node that can lead to the largest yield for the task. This process is repeated until all tasks of job j have been allocated to nodes, if possible.

A clear weakness of this algorithm is its admission policy. If a short-running job is submitted to the cluster but cannot be executed immediately due to memory constraints, it is postponed. Since we assume no knowledge of job processing times, there is no way to correlate how long a job is postponed with its processing time. A job could thus be postponed for an arbitrarily long period of time, leading to unbounded maximum stretch. The only way to circumvent this problem is to force the admission of all newly submitted jobs. This can be accomplished by pausing (via preemption) and/or moving (via migration) tasks of currently running jobs.

We define two variants of Greedy that make use of the priority function as defined previously. GreedyP operates like Greedy except that it can pause some running jobs in favor of newly submitted jobs. To do so, GreedyP goes through the list of running jobs in order of increasing priority and marks them as candidates for pausing until the incoming job could be started if all these candidates were indeed paused. It then goes through the list of marked jobs in decreasing order of priority and determines for each whether it could instead be left running due to sufficient available memory. Then, running jobs that are still marked as candidates for pausing are paused, and the new job is started. GreedyPM extends GreedyP with the capability of moving rather than pausing running jobs. This is done by trying to reschedule jobs selected for pausing in order of their priority using Greedy.

4.3 Task Mapping as Vector Packing

An alternative to the Greedy approach is to compute a global solution from scratch and then preempt and/or migrate tasks as necessary to implement that solution. As we have two resource dimensions (CPU and memory), our resource allocation problem is related to a version of bin packing, known as two-dimensional *vector packing*. One important difference between our problem and vector packing is that our jobs have fluid CPU needs. This difference can be addressed as follows: Consider a fixed value of the yield, Y, that must be achieved for all jobs. Fixing Y amounts to transforming all CPU needs into *CPU requirements*: simply multiply each CPU need by Y. The problem then becomes exactly vector packing and we can apply

a preexisting heuristic to solve it. We use a binary search on Y to find the highest yield for which the vector packing problem can be solved (our binary search has an accuracy threshold of 0.01).

In previous work [35] we developed an algorithm based on this principle called MCB8. It makes use of a two-dimensional vector packing heuristic based on that described by Leinberger et al. in [37]. The algorithm first splits the jobs into two lists, one containing all the jobs with higher CPU requirements than memory requirements. Each list is then sorted by non-increasing order of the largest requirement. The algorithm described in [37] uses the sum of the two resource dimensions to sort the jobs. We found experimentally that, for our problem and the two dimensional case, using the maximum performs marginally better [35].

Initially the algorithm assigns the first task of the first job in one of the lists (picked arbitrarily) to the first node. Subsequently, it searches, in the list that goes against the current imbalance, for the first job with an unallocated task that can fit on the node. For instance, if the node's available memory exceeds its available CPU resource (in percentage), the algorithm searches for a job in the list of memory-intensive jobs. The rationale is, on each node, to keep the total requirements of both resources in balance. If no task from any job in the preferred list fits on the node, then the algorithm considers jobs from the other list. When no task of any job in either list can fit on the node, the algorithm repeats this process for the next node.

If all tasks of all jobs can be assigned to nodes in this manner then resource allocation is successful. In the event that the MCB8 algorithm cannot find a valid allocation for all of the jobs currently in the system at any yield value, it removes the lowest priority job from consideration and tries again.

Limiting Migration

The preemption or migration of newly-submitted short-running jobs can lead to poor performance. To mitigate this behavior, we introduce two parameters. If set, the MINFT parameter (respectively, the MINVT parameter), stipulates that jobs whose flow-times (resp., virtual times) are smaller than a given bound may be paused in order to run higher priority jobs, but, if they continue running, their current node mapping must be maintained. Jobs whose flow-times (resp., virtual times) are greater than the specified bound may be moved as previously. Migrations initiated by GreedyPM are not affected by these parameters.

4.4 When to Compute New Task Mappings

So far, we have not stated *when* our task mapping algorithms should be invoked. The most obvious choice is to apply them each time a new job is submitted to the system and each time some resources are freed due to a job completion. The MCB8 algorithm attempts a global optimization and, thus, can (theoretically) "reshuffle" the whole mapping each time it is invoked[1]. One may thus fear that applying MCB8 at each job submission could lead to a prohibitive number of preemptions and migrations. On the contrary, Greedy has low overhead and the addition of a new job should not be overly disruptive to currently running jobs. The counterpart is that Greedy may generate allocations that use cluster resources inefficiently. For both of these reasons we consider variations of the algorithms that: upon job submission, either do nothing or apply Greedy, GreedyP, GreedyPM, or MCB8; upon job completion, either do nothing or apply Greedy or MCB8; and either apply or do not apply MCB8 periodically.

4.5 Algorithm Naming Scheme

We use a multi-part scheme for naming our algorithms, using '/' to separate the parts. The first part corresponds to the policy used for scheduling jobs upon submission, followed by a '*' if jobs are also scheduled opportunistically upon job completion (using MCB8 if MCB8 was used on submission, and Greedy if Greedy, GreedyP, or GreedyPM was used). If the algorithm applies MCB8 periodically, the second part contains "per". For example, the GreedyP */per algorithm performs a Greedy allocation with preemption upon job submission, opportunistically tries to start currently paused jobs using Greedy whenever a job completes, and periodically applies MCB8. Overall, we consider the 13 combinations shown in Table 1 (the 14th row of the table is explained in Section 4.7). Parameters, such as MINVT or MINFT, are appended to the name of the algorithm if set (e.g., MCB8 */per/MINFT=300).

[1] In practice this does not happen because the algorithm is deterministic and always considers the tasks and the nodes in the same order.

Table 1: DFRS Scheduling Algorithms

Name	Action		
	on submission	on completion	periodic
Greedy *	Greedy	Greedy	none
GreedyP *	GreedyP	Greedy	none
GreedyPM *	GreedyPM	Greedy	none
Greedy/per	Greedy	none	MCB8
GreedyP/per	GreedyP	none	MCB8
GreedyPM/per	GreedyPM	none	MCB8
Greedy */per	Greedy	Greedy	MCB8
GreedyP */per	GreedyP	Greedy	MCB8
GreedyPM */per	GreedyPM	Greedy	MCB8
MCB8 *	MCB8	MCB8	none
MCB8/per	MCB8	none	MCB8
MCB8 */per	MCB8	MCB8	MCB8
/per	none	none	MCB8
/stretch-per	none	none	MCB8-stretch

4.6 Resource Allocation

Once tasks have been mapped to nodes one has to decide on a CPU allocation for each job (all tasks in a job are given identical CPU allocations). All previously described algorithms use the following procedure: First all jobs are assigned yield values of $1/\max(1,\Lambda)$, where Λ is the maximum CPU load over all nodes. This maximizes the minimum yield given the mapping of tasks to nodes. After this step there may be remaining CPU resources on some of the nodes, which can be used for further improvement (without changing the mapping). We use two different approaches to exploit remaining resource fractions.

Average Yield Optimization

Once the task mapping is fixed and the minimum yield maximized, we can write a rational linear program to find a resource allocation that maximizes the average yield under the constraint that no job is given a yield lower than the maximized minimum:

$$\begin{cases} \text{MAXIMIZE } \displaystyle\sum_{j\in\mathcal{J}} y_j \text{ UNDER THE CONSTRAINTS} \\ \text{(2a)} \quad \forall j \in \mathcal{J} \quad \dfrac{1}{\max(1,\Lambda)} \leq y_j \leq 1; \\ \text{(2b)} \quad \forall i \in \mathcal{P}, \quad \displaystyle\sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{T}_j} y_j e_k^i c_j \leq 1. \end{cases} \tag{2}$$

We reuse the notation of Section 3. We use $e_k^i \in \{0,1\}$ to indicate whether task k of job j is mapped on node i (for any k, only one of the e_k^i is non-zero). Since the mapping is fixed, the e_k^i's are constants. Finally, y_j denotes the yield of job j. Linear program (2) states that the yield of any job is no smaller than the optimal minimum yield (Constraint (2a)), and that the computational power of a node cannot be exceeded (Constraint (2b)). Algorithms that use this optimization have "OPT=AVG" as an additional part of their names.

Max-min Yield Optimization

As an alternative to maximizing the average yield, we consider the iterative maximization of the minimum yield. At each step the minimum yield is maximized using the procedure described at the beginning of Section 4.6. Those jobs whose yield cannot be further improved are removed from consideration, and the minimum is further improved for the remaining jobs. This process continues until no more jobs can

be improved. This type of max-min optimization is commonly used to allocate bandwidth to competing network flows [38, Chapter 6]. Algorithms that use this optimization have "OPT=MIN" as an additional part of their names.

4.7 Optimizing the Stretch Directly

All algorithms described thus far optimize the minimum yield as a way to optimize the maximum stretch. We also consider a variant of /per, called /stretch-per, that periodically tries to minimize the maximum stretch directly, still assuming no knowledge of job processing times. The algorithm it uses, called MCB8-stretch, can only be applied periodically as it is based on knowledge of the time between scheduling events. It follows the same general procedure as MCB8 but with the following differences. At scheduling event i, the best estimate of the stretch of job j is the ratio of its flow time (time since submission), $ft_j(i)$, to its virtual time, $vt_j(i)$: $\hat{S}_j(i) = ft_j(i)/vt_j(i)$. Assuming that the job does not complete before scheduling event $i+1$, then $\hat{S}_j(i+1) = ft_j(i+1)/vt_j(i+1) = (ft_j(i)+T)/(vt_j(i)+y_j(i)\times T)$, where T is the scheduling period and $y_j(i)$ is the yield that /stretch-per assigns to job j between scheduling events i and $i+1$. Similar to the binary search on the yield, here we do a binary search to minimize $\hat{S}(i+1) = \max_j \hat{S}_j(i+1)$. At each iteration of the search, a target value $\hat{S}(i+1)$ is tested. From $\hat{S}(i+1)$ the algorithm computes the yield for each job j by solving the above equation for $y_j(i)$ (if, for any job, $y_j(i) > 1$, then $\hat{S}(i+1)$ is infeasible and the iteration fails). At that point, CPU requirements are defined and MCB8 can be applied to try to produce a resource allocation for the target value. This is repeated until the lowest feasible value is found. Since the stretch is an unbounded positive value, the algorithm actually performs a binary search over the inverse of the stretch, which is between 0 and 1. If MCB8-stretch cannot find a valid allocation for any value, then the job with lowest priority is removed from consideration and the search is re-initiated. Once a mapping of jobs to nodes has been computed each task is initially assigned a CPU allocation equal to the amount of resources it needs to reach the desired stretch. For the resource allocation improvement phase we use algorithms similar to those described in Section 4.6, except that the first (OPT=AVG) minimizes the average stretch and the second (OPT=MAX) iteratively minimizes the maximum stretch.

5 Simulation Methodology

5.1 Discrete-Event Simulator

We have developed a discrete event simulator that implements our scheduling algorithms and takes as input a number of nodes and a list of jobs. Each job is described by a submit time, a number of tasks, one CPU need and one memory requirement specification (since all tasks within a job have the same needs and requirements), and a processing time. Jobs are allocated shares of memory and CPU resources on simulated compute nodes. As stated previously in Section 2.1, the use of VM technology allows the CPU resources of a (likely multi-core) node to be shared precisely and fluidly as a single resource [22]. Thus, for each simulated node, the total amount of allocated CPU resource is simply constrained to be less than or equal to 100%. However, when simulating a multi-core node, 100% CPU resource utilization can be reached by a single task only if that task is CPU-bound and is implemented using multiple threads (or processes). A CPU-bound sequential task can use at most $100/n\%$ of the node's CPU resource, where n is the number of processor cores.

The question of properly accounting for preemption and migration overheads is a complicated one. For this reason in each simulation experiment we assume this overhead to be 5 minutes of wall clock time, whatever the job's characteristics and the number of its tasks being migrated, which is justifiably high [2]. We call this overhead the *rescheduling penalty*. In the real world there are facilities that can allow for the live migration of a running task between nodes [40], but to avoid introducing additional complexity we make the pessimistic assumption that all migrations are carried out through a pause/resume mechanism.

[2] Consider a 128-task job with 1 TB total memory, or 8 GB per task (our simulations are for a 128-node cluster). Current technology affords aggregate bandwidth to storage area networks up to tens of GB/sec for reading and writing [39]. Conservatively assuming 10 GB/sec, moving this job between node memory and secondary storage can be done in under two minutes.

Note that none of the scheduling algorithms are aware of this penalty. Based on preliminary results, we opt for a default period equal to twice the rescheduling penalty for all periodic algorithm, i.e., 10 minutes. We study the impact of the duration of the period in Section 6.4.2. The MINFT and MINVT parameters for MCB8 and MCB8-stretch are evaluated using time bounds equal to one and two penalties (i.e., 300s and 600s).

5.2 Batch Scheduling Algorithms

We consider two batch scheduling algorithms: FCFS and EASY. FCFS, often used as a baseline comparator, holds incoming jobs in a queue and assigns them to nodes in order as nodes become available. EASY [41], which is representative of production batch schedulers, is similar to FCFS but enables backfilling to reduce resource fragmentation. EASY gives the first job in the queue a reservation for the earliest possible time it would be able to run with FCFS, but other jobs in the queue are scheduled opportunistically as nodes become available, as long as they do not interfere with the reservation for the first job. EASY thus improves on FCFS by allowing small jobs to run while large jobs are waiting for a sufficiently large number of nodes. A drawback of EASY is that it requires estimations of job processing times. In all simulations we conservatively assume that EASY has perfect knowledge of job processing times. While this seems a best-case scenario for EASY, studies have shown that for some workloads some batch scheduling algorithms can produce better schedules when using non-perfectly accurate processing times (see the discussion in [42] for more details). In those studies the potential advantage of using inaccurate estimates is shown to be relatively small, while our results show that our approach outperforms EASY by orders of magnitude. Our conclusions thus still hold when EASY uses non-accurate processing time estimates.

5.3 Workloads

5.3.1 Real-World Workload

We perform experiments using a real-world workload from a well-established online repository [1]. Most publicly available logs provide standard information such as job arrival times, start time, completion time, requested duration, size in number of nodes, etc. For our purpose, we need to quantify the fraction of the resource allocated to jobs that are effectively used. We selected the "cleaned" version of the HPC2N workload from [1], which is a 182-week trace from a 120-node dual-core cluster running Linux that has been scrubbed to remove workload flurries and other anomalous data that could skew the performance comparisons on different scheduling algorithms [43]. A primary reason for choosing this workload was that it contains almost complete information regarding memory requirements, while other publicly available workloads contain no or little such information.

The HPC2N workload required some amount of processing for use in our simulation experiments. First, job per-processor memory requirements were set as the maximum of either requested or used memory as a fraction of the system memory of 2GB, to a minimum of 10%. Of the 202,876 jobs in the trace, only 2,142 ($\sim 1\%$) did not give values for either used or requested memory and so were assigned the lower bound of 10%. Second, the swf file format [1] contains information about the required number of "processors," but not the number of tasks, and so this value had to be inferred. For jobs that required an even number of processors and had a per-processor memory requirement less than 50% of the available node memory, we assumed that the job used a number of multi-threaded tasks equal to half the number of processors. In this case, we assume that each task has a CPU need of 100% (saturating the two cores of a dual-core node) and the memory requirement was doubled from its initial per-processor value. For jobs requiring an odd number of processors or more than 50% of the available node memory per processor, we assumed that the number of tasks was equal to the number of processors and that each of these tasks had a CPU need of 50% (saturating one core of a dual-core mode). Since we assume CPU-bound tasks, performance degradation due to CPU resource sharing is maximal. Consequently, our assumptions are detrimental to our approach and should benefit batch scheduling algorithms. We split the HPC2N workload into week-long segments, resulting in 182 different experimental scenarios.

5.3.2 Synthetic Workloads

We also use synthetic workloads based on the model by Lublin and Feitelson [44], augmented with additional information as described hereafter. There are several reasons for preferring synthetic workloads to real workloads for this type of relative performance evaluation: Real workloads often do not contain all of the information that we require. Further, real traces may be misleading, or lacking in critical information, such as system down times that might affect jobs running on the system [44]. Also, a real workload trace only provides a single data point, and may not be generally representative [45]. That is, the idiosyncrasies of a trace from one site may make it inappropriate for evaluating or predicting performance at another site [46]. Further, a real workload trace is the product of an existing system that uses a particular scheduling policy, and so may be biased or affected by that policy, while synthetic traces can provide a more neutral environment [46]. Finally, real workloads may contain spurious or anomalous events like user flurries that can confound evaluations of the performance of scheduling algorithms [43,47]. In fact, long workload traces often contain such events, and including them can seriously impact relative performance evaluation [48].

For the synthetic workloads we arbitrarily assume quad-core nodes, meaning that a sequential task would use at most 25% of a node's CPU resource. Due to the lack of real-world data and models regarding the CPU needs of HPC jobs, we make pessimistic assumptions similar to those described in the previous section. We assume that the task in a one-task job is sequential, but that all other tasks are multi-threaded. We assume that all tasks are CPU-bound: CPU needs of sequential tasks are 25% and those of other tasks are 100%.

The general consensus is that there is ample memory available for allocating multiple HPC job tasks on the same node [3–5], but no explicit model is available in the literature. We opt for a simple model suggested by data in Setia et al. [2]: 55% of the jobs have tasks with a memory requirement of 10%. The remaining 45% of the jobs have tasks with memory requirements $10 \times x\%$, where x is an integer value uniformly distributed over $\{2,\ldots,10\}$.

We generated 100 distinct traces of 1,000 jobs assuming a 128-node cluster. The time between the submission of the first job and the submission of the last job is on the order of 4-6 days. To study the effect of the load on our algorithms, we multiplied the job inter-arrival times in each generated trace by 9 computed constants, obtaining 9 new traces with identical job mixes but offered load [3], or *load*, levels of 0.1 to 0.9 in increments of .1. We thus have 900 *scaled* traces.

6 Experimental Results

6.1 Stretch Results

For a given problem instance, and for each algorithm, we define the *degradation from bound* as the ratio between the maximum stretch achieved by the algorithm on the instance and the theoretical lower bound on the optimal maximum stretch obtained in Section 3.1. A lower value of this ratio thus denotes better performance. We report average and standard deviation values computed over sets of problem instances, i.e., for each of our set of traces. We also report maximum values, i.e., result for the "worst trace" for each algorithm.

Results are shown in Table 2 for the FCFS and EASY batch scheduling algorithms and for 18 of our proposed algorithms. Recall that Table 1 lists 14 general combinations of mechanisms for mapping tasks to processors. All these combinations can use either OPT=AVG or OPT=MIN to compute resource allocations once a mapping has been determined. Furthermore, the last 11 combinations in Table 1 use the MCB8 algorithm and thus can use MINFT=300, MINFT=600, MINVT=300, MINVT=600, or no mechanism to limit task remapping. Therefore, the total number of potential algorithms is $3 \times 2 + 11 \times 2 \times 5 = 116$. However, the full results (see Appendix A) show that on average OPT=MIN is never worse and often slightly better than OPT=AVG. Consequently, we present results only for algorithms that use OPT=MIN. Furthermore, we found that among the mechanisms for limiting task remapping, MINVT is always better than MINFT, and slightly better with the larger 600s bound. Accordingly we also exclude algorithms that use MINFT, and algorithms that use MINVT with a 300s bound. Table 2 presents results for all 9 greedy combinations with no mechanism for limiting remapping, and results for 4 selected greedy combinations with MINVT=600, as explained hereafter. It also presents results for the 3 MCB8 combinations, for the /per algorithm, and for

Table 2: Degradation from bound results for all three trace sets, with a 5-minute rescheduling penalty

Algorithms	Real-world trace			Unscaled synthetic traces			Scaled synthetic traces		
	Degradation from bound			Degradation from bound			Degradation from bound		
	avg.	std.	max	avg.	std.	max	avg.	std.	max
FCFS	3,578.5	3,727.8	21,718.4	5,457.2	2,958.5	15,102.7	5,869.3	2,789.1	17,403.3
EASY	3,041.9	3,438.0	21,317.4	4,955.4	2,730.6	14,036.8	5,262.0	2,588.9	14,534.1
Greedy */OPT=MIN	949.8	1,828.5	11,778.4	2,435.0	2,285.6	11,229.9	3,204.3	2,517.5	19,129.2
GreedyP */OPT=MIN	13.5	68.0	819.2	37.5	156.0	1,204.9	115.7	644.0	10,354.2
GreedyPM */OPT=MIN	13.8	68.2	819.2	33.8	154.0	1,321.7	124.0	673.5	9,598.8
Greedy/per/OPT=MIN	28.3	24.9	163.7	30.1	10.2	58.1	29.3	14.3	153.2
GreedyP/per/OPT=MIN	18.5	18.6	152.4	20.1	7.3	38.1	17.8	9.6	84.6
GreedyPM/per/OPT=MIN	18.4	18.8	158.7	20.2	7.3	38.1	17.9	9.8	93.0
Greedy */per/OPT=MIN	24.3	15.9	81.6	30.4	9.7	65.7	29.1	12.3	101.4
GreedyP */per/OPT=MIN	17.9	19.6	213.5	20.3	6.8	32.0	17.9	8.6	89.9
GreedyPM */per/OPT=MIN	17.9	19.3	198.6	20.3	6.9	32.0	17.9	8.6	89.9
GreedyP/per/OPT=MIN/MINVT=600	8.9	18.9	152.4	5.9	4.5	38.1	7.3	8.5	96.8
GreedyPM/per/OPT=MIN/MINVT=600	8.8	18.9	158.7	5.9	4.5	38.1	7.3	8.1	96.8
GreedyP */per/OPT=MIN/MINVT=600	6.9	14.2	149.3	4.9	2.9	19.2	6.1	6.3	103.5
GreedyPM */per/OPT=MIN/MINVT=600	6.9	14.4	149.6	4.8	2.4	13.6	6.1	5.4	90.2
MCB8 */OPT=MIN/MINVT=600	12.0	32.8	370.0	6.9	5.4	44.4	13.2	21.6	270.9
MCB8/per/OPT=MIN/MINVT=600	10.8	25.3	287.6	8.1	6.6	53.3	11.0	12.6	127.5
MCB8 */per/OPT=MIN/MINVT=600	13.6	30.2	318.9	7.8	3.9	21.9	12.2	15.3	195.7
/per/OPT=MIN/MINVT=600	105.0	445.6	5,011.9	43.0	19.7	134.7	40.4	25.0	238.3
/stretch-per/OPT=MAX/MINVT=600	105.0	445.6	5,011.9	43.0	19.6	134.7	40.2	24.8	236.9

the /stretch-per algorithm. All these are with MINVT=600. We leave results without MINVT=600 for the 3 MCB8 combinations out of the table because these algorithms perform very poorly. Indeed, as they apply MCB8 upon each job arrival, they lead to inordinate amounts of remapping and thus are more than one order of magnitude further away from the bound than when MINVT=600 is used. For /per and /stretch-per, the addition of MINVT=600 does not matter since the scheduling period is no shorter than 600s. We are left with the 18 algorithms in the table, which we discuss hereafter.

The results in Table 2 are mostly consistent across our three sets of traces, with a few exceptions. Expectedly, EASY outperforms FCFS. The key observation is that EASY and FCFS are outperformed by our proposed algorithms by several orders of magnitude, thereby showing that DFRS is an attractive alternative to batch scheduling.

The table shows results for 4 groups of greedy algorithms. In the first group are algorithms that do not apply MCB8 periodically (i.e., those without "per " in their names). On average, these algorithms lead to results poorer than the 6 algorithms in the next 2 groups, which do apply MCB8 periodically, on the synthetic traces. For real-world traces, we see that GreedyP * and GreedyPM * lead to the best average case results for algorithms from these 3 groups. For all traces, however, the purely greedy algorithms lead to standard deviation and maximum values that are orders of magnitude larger than those obtained by the greedy algorithms that also make use of MCB8 periodically. We conclude that applying MCB8 periodically is beneficial for greedy algorithms. The results also show that GreedyP is better than Greedy, demonstrating the benefits of preemption. However, the use of migration by GreedyPM does not lead to significant further improvement and can even lead to a slight decrease in performance. It turns out that the jobs migrated in the GreedyPM approach are often low priority and thus have a high probability of being preempted at an upcoming scheduling event anyway. Finally, by comparing the results for the 2nd and 3rd groups of greedy algorithms, we see that scheduling jobs opportunistically (i.e., as done by algorithms with * in their names) also seems to have limited, but generally positive, impact on performance when combined with applying MCB8 periodically.

The 4 algorithms from the last group of greedy algorithms use either GreedyP or GreedyPM, with or without opportunistic scheduling, but with the MINVT=600 feature to limit task remapping by MCB8. We see that these algorithms outperform all previously discussed algorithms on all 3 sets of traces. For these algorithms the use of opportunistic scheduling is always beneficial. In this case, GreedyP and GreedyPM lead to similar results. These algorithms are the best overall, including in terms of standard deviation.

The 3 algorithms in the next group all use MCB8 to assign tasks to processors upon job submission (and possibly completion) rather than a greedy approach. They all use MINVT=600 because without limiting task

remapping they all lead to poorer performance by orders of magnitude due to job thrashing. Overall, while these algorithms perform very well, they are not among the best.

The next algorithm, /per/OPT=MIN/MINVT=600, simply applies MCB8 periodically without taking action upon job arrival or job completion. It is outperformed by the best greedy algorithm more than 6-fold. This result confirms the notion that a scheduling algorithm should react to job submissions. The algorithm in the last row of the table, /stretch-per/OPT=MAX/MINVT=600, optimizes the stretch directly. It performs on par with /per/OPT=MIN/MINVT=600, but more than one order of magnitude worse than our best yield-based algorithm. This demonstrates that yield optimization is a good approach and that an algorithm for minimizing the stretch directly may not be achievable.

Our overall conclusion from the results in Table 2 is that, to achieve good performance, all our techniques should be combined: an aggressive greedy job admission policy with preemption of running jobs, a periodic use of the MCB8 vector-packing algorithm, an opportunistic use of resources freed upon job completion, and a grace period that prevents remapping of tasks that have just begun executing. Note that while the algorithms are executed in an online, non-clairvoyant context, the computation of the bound on the optimal performance relies on knowledge of both the release dates and processing times of all jobs. Furthermore, the bound ignores memory constraints. Nevertheless, in our experiments, our best algorithms are on average at most a factor 7 away from this loose bound. We conclude that our algorithms lead to good performance in an absolute sense.

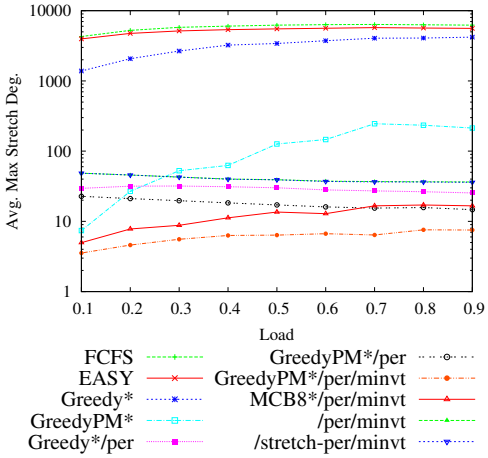


Figure 1: Average degradation from bound vs. load for selected algorithms on the scaled synthetic dataset, assuming a 5-minute rescheduling penalty. Each data point shows average values over 100 instances. All algorithms use OPT=MIN (except /stretch-per, which uses OPT=MAX) and use MINVT=600 if MINVT is specified.

Figure 1 plots average degradation factors vs. load for selected algorithms when applied to scaled synthetic workloads and using the 5-minute rescheduling penalty, using a logarithmic scale on the vertical axis. We plot only the Greedy and GreedyPM versions of the algorithms, and omit the OPT=MIN and the "=600" parts of the names so as not to clutter the caption. Most of the observations made for the results in Table 2 hold for the results in the figure. One notable feature here is that the GreedyPM */OPT=MIN algorithm performs well under low load conditions but quickly leads to poorer performance than most other algorithms as the load becomes larger than 0.3. Under low-load conditions, the periodic use of MCB8 to remap tasks is not as critical to ensure good performance. The main observation in this figure is that our best algorithm, GreedyPM */per/OPT=MIN/MINVT=600, is the best algorithm across the board regardless of the load level. It is about a factor 3 away from the bound at low loads, and a factor 7.5 away from it at high loads.

Table 3: Preemption and migration costs in terms of average bandwidth consumption, number of preemption and migration occurrences per hour, and number of preemption and migration occurrences per job. Average values over scaled synthetic traces with load ≥ 0.7, with maximum values in parentheses.

Algorithm	Bandwidth Consumption (GB / sec)		Frequency of Occurrence (# occurrences / hour)		# occurrences per job	
	pmtn	mig	pmtn	mig	pmtn	mig
EASY	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
FCFS	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Greedy */OPT=MIN	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
GreedyP */OPT=MIN	0.06 (0.17)	0.00 (0.00)	5.67 (18.00)	0.00 (0.00)	0.57 (2.04)	0.00 (0.00)
GreedyPM */OPT=MIN	0.03 (0.07)	0.02 (0.05)	2.25 (10.08)	3.69 (13.32)	0.23 (1.19)	0.36 (1.22)
Greedy/per/OPT=MIN	0.48 (1.08)	0.21 (0.60)	32.58 (83.52)	38.79 (110.52)	5.41 (21.76)	4.81 (16.17)
GreedyP/per/OPT=MIN	0.50 (1.11)	0.20 (0.60)	33.34 (85.32)	37.52 (107.64)	5.54 (22.20)	4.67 (15.76)
GreedyPM/per/OPT=MIN	0.49 (1.10)	0.21 (0.60)	32.93 (84.24)	39.21 (112.32)	5.52 (21.65)	4.85 (16.03)
Greedy */per/OPT=MIN	0.50 (1.29)	0.27 (0.66)	29.27 (84.96)	58.06 (124.56)	4.49 (22.55)	6.94 (17.65)
GreedyP */per/OPT=MIN	0.58 (1.37)	0.28 (0.65)	39.67 (97.92)	58.56 (127.08)	5.87 (24.87)	7.09 (17.91)
GreedyPM */per/OPT=MIN	0.56 (1.37)	0.29 (0.66)	35.78 (96.48)	62.95 (135.72)	5.42 (24.00)	7.59 (18.32)
Greedy */per/OPT=MIN/MINVT=600	0.49 (1.27)	0.24 (0.62)	28.08 (83.52)	51.97 (117.36)	4.29 (21.80)	6.25 (16.53)
GreedyP */per/OPT=MIN/MINVT=600	0.56 (1.36)	0.24 (0.63)	37.70 (97.20)	52.14 (119.16)	5.56 (23.88)	6.34 (16.48)
GreedyPM */per/OPT=MIN/MINVT=600	0.54 (1.34)	0.26 (0.62)	33.80 (94.32)	56.45 (127.08)	5.11 (23.23)	6.84 (16.94)
MCB8 */OPT=MIN	0.42 (1.26)	1.17 (2.36)	61.66 (230.40)	490.48 (1,005.48)	6.67 (18.83)	57.46 (90.29)
MCB8 */per/OPT=MIN	0.72 (1.15)	1.21 (2.68)	77.04 (170.28)	479.31 (1,109.52)	13.01 (28.93)	73.61 (117.15)
MCB8 */per/OPT=MIN/MINVT=600	0.54 (1.11)	0.56 (1.53)	37.94 (85.32)	194.57 (836.28)	6.57 (26.52)	22.55 (50.29)
/per/OPT=MIN	0.49 (1.07)	0.21 (0.62)	33.83 (84.24)	38.69 (111.24)	5.65 (23.23)	4.90 (16.58)
stretch-per/OPT=MAX	0.28 (0.65)	0.39 (0.81)	20.41 (45.36)	67.26 (159.48)	3.79 (16.71)	10.41 (26.96)

6.2 Algorithm Execution Time

To show that our approach produces resource allocations quickly we record the running time of MCB8, the most computationally expensive component of our algorithms by orders of magnitude. We ran the simulator using the MCB8 * algorithm, which applies MCB8 the most frequently, on a system with a 3.2GHz Intel Xeon CPU and 4GB RAM for the 100 unscaled traces generated by the Lublin model. This experiment resulted in a total of 197,808 observations. For 67.25% of these observations MCB8 produced allocations for 10 or fewer jobs in less than 0.001 seconds. The remaining observations were for 11 to 102 jobs. The average compute time was about 0.25 seconds with the maximum under 4.5 seconds. Since typical job inter-arrival times are orders of magnitude larger [44], we conclude that our best algorithms can be used in practice.

6.3 Impact of Preemptions and Migrations

The previous section shows that our algorithms wildly outperform EASY. One may, however, wonder whether network and I/O resources are not overly taxed by preemption and/or migration activity. Table 3 shows results obtained using our synthetic scaled trace data, and only for traces with load levels 0.7 or higher. The table shows averages and, in parentheses, maxima, computed over all traces for our two best algorithms from the previous section. For each algorithm, the first two result columns of the table show the overall bandwidth consumption due to preemptions and migrations in GB/sec. The next two columns show the frequency of job preemptions and migrations, and the last two columns show the number of preemptions and migrations per job. These numbers are computed by saying that a job is preempted (resp. migrated) whenever one or more of its tasks is preempted (resp. migrated).

The main observation from the results in Table 3 is that the total bandwidth consumption is reasonable. The two algorithms have a total average bandwidth consumption under 0.80 GB/sec. Even accounting for maximum bandwidth consumption, i.e., for the trace that causes the most traffic due to preemptions and migrations, bandwidth consumptions are under 2.0 GB/sec. Such numbers represent only a small fraction of the bandwidth capacity of current interconnect technology for cluster platforms. The numbers of preemptions per hour show that under 40 preemptions and under 60 migrations occur each hour, with each job being preempted under 6 times and migrating under 7 times during its lifetime, on average. Again, in light of the good performance achieved by these algorithms, these numbers are reasonable in spite of our conservative 5-minute rescheduling penalty.

Results are similar for other algorithms that use the greedy approach and apply MCB8 periodically. The algorithms that use the greedy approach and do not employ MCB8 periodically expectedly lead to lower numbers of migrations and preemptions per hour (under 10 for each, on average). The algorithms that use the MCB8 algorithm at each job arrival, and possibly at each job completion, lead to higher number of preemptions per hour and, more noticeably, much higher numbers of migrations per hour (above 120 and up to 490). These algorithms suffer from job thrashing, which has been identified in the previous section as the main cause for their poor performance. This phenomenon is also seen in per-job numbers. While all other algorithms migrate a job less than 8 times on average, these algorithms migrate a job 14 times on average and up to 73 times in the worst case.

Overall, the bandwidth consumption due to preemption/migration and the number of preemption/migration events reported in Table 3 show that our algorithms can be put in practice for real-world platforms. Complete tables detailing the preemption and migration costs for all algorithms are provided in Appendix A.

6.4 Platform Utilization

In this section we investigate how our best algorithms, in terms of maximum stretch, compare to EASY in terms of platform utilization. We introduce a new metric, called *underutilization*. We contend that this metric helps quantify schedule quality in terms of utilization, while remaining agnostic to conditions that can confound traditional metrics, as explained hereafter.

6.4.1 Measuring System Underutilization

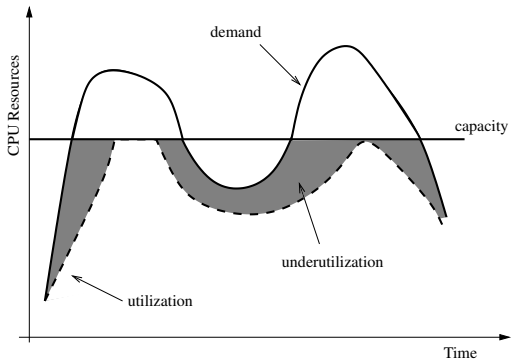


Figure 2: Illustration of underutilization

We have shown that DFRS algorithms are capable of dramatically improving the performance delivered to jobs when compared with traditional batch scheduling approaches. However, job performance is not the sole concern for system administrators. In fact, a common goal is to maximize platform utilization so as to both improve throughput and justify the costs associated with maintaining a cluster. Metrics used to evaluate machine utilization include throughput, daily peak instantaneous utilization, and average instantaneous utilization over time. However, these metrics are not appropriate for open, online systems as they are highly dependent on the arrival process and the requirements of jobs in the workload [47]. Another potential candidate, the makespan, i.e., the amount of time elapsed between the submission of the first job in the workload and the completion of last job, suffers from the problem that a very short job may be submitted at the last possible instant, resulting in all the scheduling algorithms having (nearly) the same makespan [47].

Instead, we argue that the quality of a scheduling algorithm (not considering fairness to jobs) should be judged based on how well it meets demand over the course of time, bounded by the resource constraints of the system. We call our measure of this quantity the *underutilization* and define it as follows. For a fixed set of nodes $\mathcal{P}$, let $D^{\sigma}_{\mathcal{J}}(t)$ be the total CPU demand (i.e., sum of CPU needs) by jobs from $\mathcal{J}$ that have been submitted but have not yet completed at time t when scheduled using algorithm σ, and let $u^{\sigma}_{\mathcal{J}}(t)$ represent the total system utilization at time t (i.e., sum of allocated CPU fractions) under the same conditions. The underutilization of a system (assuming that all release dates are ≥ 0) is given by

Table 4: Average normalized underutilization for selected algorithms on all datasets

Algorithm	Real-world trace	Unscaled synthetic traces	Scaled synthetic traces
EASY	0.064	0.349	0.384
GreedyP */per/OPT=MIN/MINVT=600	0.344	0.497	0.607
GreedyPM */per/OPT=MIN/MINVT=600	0.344	0.497	0.608

$\int_0^\infty \min\{|\mathcal{P}|, D^\sigma_\mathcal{J}(t)\} - u^\sigma_\mathcal{J}(t)dt$. Note that $u^\sigma_\mathcal{J}(t)$ is constrained to always be less than both $|\mathcal{P}|$ and $D^\sigma_\mathcal{J}(t)$, and that $D^\sigma_\mathcal{J}(t) = 0$ outside of the time span between when the first job is submitted and the last one completes.

Figure 2 provides a visual explanation. The horizontal axis represents time while the vertical axis shows CPU resources. The solid horizontal line shows the total system CPU capacity, $|\mathcal{P}|$. The solid curve shows the total instantaneous demand, $D^\sigma_\mathcal{J}(t)$. The dashed curve shows the total instantaneous utilization over time, $u^\sigma_\mathcal{J}(t)$. The gray area, bounded above by the minimum of CPU capacity and CPU demand, and below by CPU utilization, represents what we call the *underutilization* over the given time period.

Essentially, underutilization represents a cumulative measure over time of computational power that could, at least theoretically, be exploited, but that is instead sitting idle (or being used for non-useful work, such as preemption or migration). Thus, lower values for underutilization are preferable to higher values. As this quantity depends on total workload demand, which can vary considerably, when combining results over a number of workloads we consider the normalized underutilization, or the underutilization as a fraction of the total resources required to execute the workload. A normalized underutilization value of 1.0 would mean that, over the course of executing a workload, another scheduling algorithm could have used an average of twice as much CPU resource at any given time (ignoring memory and migration constraints). We contend that for a fixed platform algorithms that do a better job of allocating resources will tend to have smaller values for normalized underutilization in the average case on a given set of workloads. It is important to note that normalized underutilization will also vary with platform and workload characteristics, and thus it should not be taken as an absolute measure of algorithm efficiency.

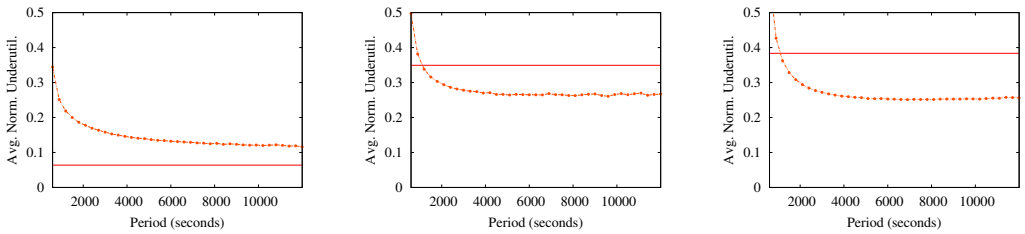
6.4.2 Experimental Results

In this section we consider only the EASY batch scheduling algorithm and the two best algorithms identified in Section 6.1: GreedyP */per/OPT=MIN/MINVT=600 and GreedyPM */per/OPT=MIN/MINVT=600. Table 4 shows average normalized underutilization results for our three sets of traces. We can see that our two algorithms lead to similar underutilization, and that they both lead to higher underutilization than EASY. This is particularly striking for the real-world HPC2N trace, for which EASY leads to underutilization under 7% while our algorithms are above 34%.

While EASY scores better on this initial comparison, there are several factors to consider. The first is that the very low underutilization shown by EASY against the real-world trace may be partially due to an artifact of either user or system behavior (e.g., the system from which the log was harvested also made use of the EASY scheduler). Also for the real trace, the fact that we arbitrarily split the entire HPC2N trace into 182 week-long periods may be a factor. A simulation for the full trace yields a normalized underutilization of 8% for EASY and of 10% for our algorithms.

Still, our proposed algorithms also perform worse in terms of underutilization on the synthetic traces, which should be free from both of the above problems. We hypothesize that this lower efficiency is caused by time spent doing preemption and migration. Recall that all our algorithms use a 600 second period, equal to the rescheduling penalty, by default. By increasing the period, within reasonable bounds, one can then hope to decrease underutilization. The trade-off, however, is that the maximum stretch may be increased.

Figure 3 shows average normalized underutilization results on our three sets of traces for EASY and the GreedyPM */per/OPT=MIN/MINVT=600 algorithm (results for GreedyP */per/OPT=MIN/MINVT=600 are similar). For each set, we plot the average normalized underutilization versus the period. In all graphs the period varies from 600 to 12,000 seconds, i.e., from 2x to 20x the rescheduling penalty. We do not use a period equal to the penalty as for some traces it can result in a situation where no job ever makes any progress due to thrashing. In all graphs normalized underutilization is shown to decrease steadily.



(a) Real-world traces (b) Unscaled synthetic traces (c) Scaled synthetic traces

Figure 3: Average normalized underutilization vs. period for EASY (solid) and GreedyPM */per/OPT=MIN/MINVT=600 (dots)

Additional results provided in Appendix B show that for extremely large periods (over 15,000 seconds for the synthetic traces) underutilization expectedly begins to increase. For the two sets of synthetic traces, as the period becomes larger than 900s (i.e., 1.5x the rescheduling penalty), our algorithm achieves better average normalized underutilization than EASY. For the real-world trace, our algorithm achieves higher values than EASY regardless of the period: EASY achieves low values at around 6.4%, while our algorithm plateaus at around 11.8%, before slowly starting to increase again for larger periods (see Appendix B).

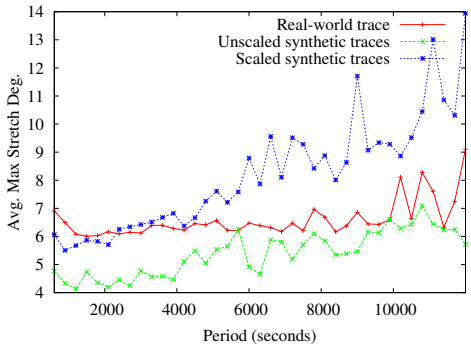


Figure 4: Maximum stretch degradation from bound vs. scheduling period for GreedyPM */per/OPT=MIN/MINVT=600 for all three trace sets

Figure 4 shows the average maximum stretch for our algorithm as the period increases, for each set of traces. The main observation is that the average maximum stretch degradation increases slowly as the period increases. The largest increase is seen for the scaled synthetic traces, for which a 20-fold increase of the period leads to an increase of the maximum stretch degradation by less than a factor 3. Results for the unscaled synthetic traces and the real-world trace show much smaller increases (by less than a factor 1.5 and a factor 2, respectively). Recall from Section 6.1 that EASY leads to maximum stretch degradation orders of magnitude larger than our algorithms. Consequently, increasing the period significantly, i.e., up to 20x the rescheduling penalty, still allows our algorithms to outperform EASY by at least two orders of magnitude on average.

We conclude that our algorithms can outperform EASY by orders of magnitude in terms of maximum stretch, and lead to only slightly higher or significantly lower underutilization provided the period is set appropriately. Our results indicate that picking a period roughly between 5x and 20x the rescheduling penalty leads to good results. Our approach is thus robust to the choice of the period, and in practice a period of, say, 1 hour, is appropriate. Results summarized in Figure 9 of Appendix B show that with a 1-hour period the bandwidth consumption due to preemptions and migrations is roughly 4 times lower than that reported in Section 6.3, e.g., under 0.2GB/sec on the average when considering the scaled synthetic traces with load values ≥ 0.7. As a conclusion, we recommend using the GreedyPM */per/OPT=MIN/MINVT=600 algorithm with a period equal to 10 times the rescheduling penalty.

7 Related Work

Gang scheduling [14] is the classical approach to time-sharing cluster resources. In gang scheduling, tasks in a parallel job are executed during the same synchronized time slices across cluster nodes. This requires distributed synchronized context-switching, which may impose significant overhead and thus long time slices, although solutions have been proposed [49]. In this work we simply achieve time-sharing in an uncoordinated and low-overhead manner via VM technology.

A second drawback of gang scheduling is the memory pressure problem, i.e., the overhead of swapping to disk [3]. In our approach we completely avoid swapping by enforcing the rule that a task may be assigned to a node only if physical memory capacity is not exceeded. This precludes some time sharing when compared to standard gang scheduling. However, this constraint is eminently sensible given the performance penalties associated with swapping to disk and the observation that many jobs in HPC workloads use only a fraction of physical memory [2–5].

Other works have explored the problem of scheduling jobs without knowledge of their processing time. The famous "scheduling in the dark" approach [33] shows that, in the absence of knowledge, giving equal resource shares to jobs is theoretically sound. We use the same approach by ensuring that all jobs achieve the same yield. Our problem is also related to the thread scheduling done in operating system kernels, given that thread processing times are unknown. Our work differs in that we strive to optimize a precisely defined objective function.

Our work is also related to several previous works that have explored algorithmic issues pertaining to bin packing and/or multiprocessor scheduling. There are obvious connections to *fully dynamic* bin packing, a formulation where items may arrive or depart at discrete time intervals and the goal is to minimize the maximum number of bins required while limiting re-packing, as studied by Ivkovic and Lloyd [50]. Coffman studies bin stretching, a version of bin packing in which a bin may be stretched beyond its normal capacity [51]. Epstein studies the online bin stretching problem as a scheduling problem with the goal of minimizing makespan [52].

Our scheduling problem is strongly related to vector packing, i.e., bin packing with multi-dimensional items and bins. Vector packing has been studied from both a theoretical standpoint (i.e., guaranteed algorithms) and a pragmatic one (i.e., efficient algorithms). In this work we employ an algorithm based on a particular vector packing algorithm, MCB (Multi-Capacity Bin Packing), proposed by Leinberger et al. [37]. We refer the reader to [34] for an extensive review of the literature on vector packing algorithms.

Yossi Azar has studied online load balancing of temporary tasks on identical machines with assignment restrictions [53]. Each task has a weight and a duration. The weight is known when the task arrives, but the duration is not known until the task completes. The problem therein is to assign incoming tasks to nodes permanently so that the maximum load is minimized over both nodes and time, which is related to maximizing the minimum yield.

Finally, previous works have explored the use of VM technology in the HPC domain. Some of the potential benefits of cluster virtualization include increased reliability [54], load balancing, and consolidation to reduce the number of active nodes [12]. Current VM technology also allows for preemption and checkpointing of MPI applications [54], as assumed in this work. The broad consensus is that VM overhead does not represent a barrier to mainstream deployment [55]. Additional research has shown that the performance impact on MPI applications is minimal [56] and that cache interference effects do not cause significant performance degradation in commonly-used libraries such as LAPACK and BLAS [57].

Several groups in academia and the private sector are currently working on platforms for centralized control of virtualized resources in a distributed environment [11,12,15–19,58–61]. These platforms generally allow a central controller to create VMs, specify resource consumption levels, migrate VMs between nodes, suspend running instances to disk, and, when necessary, delete unruly instances. We base our model on such a system and the capabilities it offers. The Entropy system recently developed by Hermenier et al. [12] in fact implements all of the basic system capabilities that we propose to exploit as well as its own set of resource allocation algorithms, but their approach is based on searching for solutions to an NP-complete constraint satisfaction problem, while our approach is to develop polynomial time heuristic algorithms for a well-defined optimization problem.

Other groups have proposed VM-enhanced scheduling algorithms, but for the most part these have been refinements of or extensions to existing schemes, such as combining best-effort and reservation based

jobs [13]. Fallenbeck et al. developed a novel solution that allows two virtual clusters to coexist on the same physical cluster, with each physical node mapped to two virtual nodes, but their implementation is meant to solve a problem specific to a particular site and does not add to the literature in terms of general scheduling algorithms [62]. The system proposed by Ruth et al. attempts to dynamically migrate tasks from "overloaded" hosts, but their definition of overloaded is vague and they do not propose a well-defined objective function [63].

8 Conclusion

In this paper we have proposed DFRS, a novel approach for job scheduling on a homogeneous cluster. We have focused on an online, non-clairvoyant scenario in which job processing times are unknown ahead of time. We have proposed several scheduling algorithms and have compared them to standard batch scheduling approaches using both real-world and synthetic workloads. We have found that several DFRS algorithms lead to dramatic improvement over batch scheduling in terms of maximum (bounded) stretch. In particular, algorithms that periodically apply the MCB8 vector packing algorithms lead to the best results. Our results also show that the network bandwidth consumption of these algorithms for job preemptions and migrations is only a small fraction of that available in current clusters. Finally, we have shown that these algorithms can lead to good platform utilization as long as the period at which MCB8 is applied is chosen within a broad range. The improvements shown in our results are likely to be larger in practice due the many conservative assumptions in our evaluation methodology.

This work opens a number of promising directions for future research. Our scheduling algorithms could be improved with a strategy for reducing the yield of long running jobs. This strategy, inspired by thread scheduling in operating systems kernels, would be particularly useful for mitigating the negative impact of long running jobs on shorter ones, thereby improving fairness. While we considered the case for HPC jobs composed of tasks with homogeneous resource requirements and needs, the techniques that we developed could easily be modified to allow for heterogeneous tasks as well (see our paper on the offline problem [34] for an expanded discussion of this issue). Also, mechanisms for implementing user priorities, such as those supported in batch scheduling systems, are needed. More broadly, a logical next step is to implement and benchmark our algorithms as part of a prototype virtual cluster management system that uses some of the resource need discovery techniques described in Section 2.1.

Acknowledgment

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A Additional Tables

Table 5: Average degradation from bound results for the real-world HPC2N workload. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
FCFS	3,578.5	3,727.8	21,718.4
EASY	3,041.9	3,438.0	21,317.4
Greedy */OPT=AVG	1,012.2	2,229.5	19,799.1
Greedy */OPT=MIN	949.8	1,828.5	11,778.4
Greedy/per/OPT=AVG	28.9	27.0	212.9
Greedy/per/OPT=AVG/MINFT=300	23.3	26.9	182.3
Greedy/per/OPT=AVG/MINFT=600	23.5	27.6	212.2
Greedy/per/OPT=AVG/MINVT=300	23.9	26.9	182.3
Greedy/per/OPT=AVG/MINVT=600	23.8	27.8	182.3
Greedy/per/OPT=MIN	28.3	24.9	163.7
Greedy/per/OPT=MIN/MINFT=300	23.4	26.0	152.0
Greedy/per/OPT=MIN/MINFT=600	23.1	24.8	152.5
Greedy/per/OPT=MIN/MINVT=300	23.5	26.9	182.8
Greedy/per/OPT=MIN/MINVT=600	23.0	25.9	152.0
Greedy */per/OPT=AVG	24.5	15.9	81.6
Greedy */per/OPT=AVG/MINFT=300	19.8	17.8	85.6
Greedy */per/OPT=AVG/MINFT=600	19.3	17.6	85.6
Greedy */per/OPT=AVG/MINVT=300	19.2	17.5	85.6
Greedy */per/OPT=AVG/MINVT=600	18.9	17.3	74.9
Greedy */per/OPT=MIN	24.3	15.9	81.6
Greedy */per/OPT=MIN/MINFT=300	19.5	17.7	85.6
Greedy */per/OPT=MIN/MINFT=600	19.1	17.5	85.6
Greedy */per/OPT=MIN/MINVT=300	18.9	17.2	85.6
Greedy */per/OPT=MIN/MINVT=600	19.0	17.4	66.1
GreedyP */OPT=AVG	20.4	116.7	1,254.2
GreedyP */OPT=MIN	13.5	68.0	819.2
GreedyP/per/OPT=AVG	18.4	18.6	152.4
GreedyP/per/OPT=AVG/MINFT=300	9.1	18.8	152.4
GreedyP/per/OPT=AVG/MINFT=600	9.0	18.7	152.4
GreedyP/per/OPT=AVG/MINVT=300	9.0	18.8	152.4
GreedyP/per/OPT=AVG/MINVT=600	8.9	18.9	152.4
GreedyP/per/OPT=MIN	18.5	18.6	152.4
GreedyP/per/OPT=MIN/MINFT=300	9.1	18.8	152.4
GreedyP/per/OPT=MIN/MINFT=600	9.0	18.9	152.4
GreedyP/per/OPT=MIN/MINVT=300	9.0	18.9	152.4
GreedyP/per/OPT=MIN/MINVT=600	8.9	18.9	152.4
GreedyP */per/OPT=AVG	17.9	19.7	213.5
GreedyP */per/OPT=AVG/MINFT=300	8.2	19.4	213.5
GreedyP */per/OPT=AVG/MINFT=600	7.7	18.5	198.8
GreedyP */per/OPT=AVG/MINVT=300	7.0	14.0	149.3
GreedyP */per/OPT=AVG/MINVT=600	6.9	14.0	149.3
GreedyP */per/OPT=MIN	17.9	19.6	213.5
GreedyP */per/OPT=MIN/MINFT=300	7.9	19.0	213.5
GreedyP */per/OPT=MIN/MINFT=600	7.5	18.0	198.8
GreedyP */per/OPT=MIN/MINVT=300	6.9	14.0	149.3
GreedyP */per/OPT=MIN/MINVT=600	6.9	14.2	149.3

Table 5: (Continued) Average degradation from bound results for the real-world HPC2N workload. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
GreedyPM */OPT=AVG	14.1	72.7	880.1
GreedyPM */OPT=MIN	13.8	68.2	819.2
GreedyPM/per/OPT=AVG	18.5	19.2	158.7
GreedyPM/per/OPT=AVG/MINFT=300	9.1	19.4	158.7
GreedyPM/per/OPT=AVG/MINFT=600	9.0	19.3	158.7
GreedyPM/per/OPT=AVG/MINVT=300	9.1	19.4	158.7
GreedyPM/per/OPT=AVG/MINVT=600	8.9	19.5	158.7
GreedyPM/per/OPT=MIN	18.4	18.8	158.7
GreedyPM/per/OPT=MIN/MINFT=300	9.2	19.2	158.7
GreedyPM/per/OPT=MIN/MINFT=600	9.1	19.3	158.7
GreedyPM/per/OPT=MIN/MINVT=300	8.9	18.8	158.7
GreedyPM/per/OPT=MIN/MINVT=600	8.8	18.9	158.7
GreedyPM */per/OPT=AVG	17.8	19.3	198.6
GreedyPM */per/OPT=AVG/MINFT=300	8.2	19.1	198.6
GreedyPM */per/OPT=AVG/MINFT=600	7.8	19.0	198.8
GreedyPM */per/OPT=AVG/MINVT=300	7.0	14.1	149.6
GreedyPM */per/OPT=AVG/MINVT=600	6.9	14.2	149.6
GreedyPM */per/OPT=MIN	17.9	19.3	198.6
GreedyPM */per/OPT=MIN/MINFT=300	8.1	19.2	198.6
GreedyPM */per/OPT=MIN/MINFT=600	7.9	19.1	198.8
GreedyPM */per/OPT=MIN/MINVT=300	6.9	14.3	149.6
GreedyPM */per/OPT=MIN/MINVT=600	6.9	14.4	149.6
MCB8 */OPT=AVG	346.3	1,223.1	13,399.7
MCB8 */OPT=AVG/MINFT=300	44.7	144.2	1,082.8
MCB8 */OPT=AVG/MINFT=600	20.6	63.9	672.0
MCB8 */OPT=AVG/MINVT=300	14.1	33.2	370.0
MCB8 */OPT=AVG/MINVT=600	12.1	32.8	370.0
MCB8 */OPT=MIN	345.6	1,241.5	13,668.8
MCB8 */OPT=MIN/MINFT=300	44.7	138.4	1,126.3
MCB8 */OPT=MIN/MINFT=600	19.1	59.2	672.0
MCB8 */OPT=MIN/MINVT=300	14.0	33.2	370.0
MCB8 */OPT=MIN/MINVT=600	12.0	32.8	370.0
MCB8/per/OPT=AVG	171.4	702.4	8,383.0
MCB8/per/OPT=AVG/MINFT=300	15.0	33.5	279.3
MCB8/per/OPT=AVG/MINFT=600	12.1	26.9	292.4
MCB8/per/OPT=AVG/MINVT=300	11.5	25.3	287.6
MCB8/per/OPT=AVG/MINVT=600	10.7	25.2	287.6
MCB8/per/OPT=MIN	169.1	691.4	8,362.3
MCB8/per/OPT=MIN/MINFT=300	15.1	34.5	309.3
MCB8/per/OPT=MIN/MINFT=600	12.1	26.9	292.4
MCB8/per/OPT=MIN/MINVT=300	11.5	25.3	287.6
MCB8/per/OPT=MIN/MINVT=600	10.8	25.3	287.6
MCB8 */per/OPT=AVG	394.5	1,562.9	17,862.0
MCB8 */per/OPT=AVG/MINFT=300	56.1	197.4	1,849.6
MCB8 */per/OPT=AVG/MINFT=600	23.6	71.3	799.3
MCB8 */per/OPT=AVG/MINVT=300	15.5	30.6	318.9
MCB8 */per/OPT=AVG/MINVT=600	13.7	30.3	318.9
MCB8 */per/OPT=MIN	389.1	1,522.1	17,313.6

Table 5: (Continued) Average degradation from bound results for the real-world HPC2N workload. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
MCB8 */per/OPT=MIN/MINFT=300	56.8	198.7	1,738.4
MCB8 */per/OPT=MIN/MINFT=600	25.1	81.4	799.3
MCB8 */per/OPT=MIN/MINVT=300	15.4	30.6	318.9
MCB8 */per/OPT=MIN/MINVT=600	13.6	30.2	318.9
/per/OPT=AVG	105.0	445.6	5,011.9
/per/OPT=AVG/MINFT=300	105.0	445.6	5,011.9
/per/OPT=AVG/MINFT=600	105.0	445.6	5,011.9
/per/OPT=AVG/MINVT=300	105.0	445.6	5,011.9
/per/OPT=AVG/MINVT=600	105.0	445.6	5,011.9
/per/OPT=MIN	105.0	445.6	5,011.9
/per/OPT=MIN/MINFT=300	105.0	445.6	5,011.9
/per/OPT=MIN/MINFT=600	105.0	445.6	5,011.9
/per/OPT=MIN/MINVT=300	105.0	445.6	5,011.9
/per/OPT=MIN/MINVT=600	105.0	445.6	5,011.9
/stretch-per/OPT=AVG	105.0	445.6	5,011.9
/stretch-per/OPT=AVG/MINFT=300	105.0	445.6	5,011.9
/stretch-per/OPT=AVG/MINFT=600	105.0	445.6	5,011.9
/stretch-per/OPT=AVG/MINVT=300	105.0	445.6	5,011.9
/stretch-per/OPT=AVG/MINVT=600	105.0	445.6	5,011.9
/stretch-per/OPT=MAX	105.0	445.6	5,011.9
/stretch-per/OPT=MAX/MINFT=300	105.0	445.6	5,011.9
/stretch-per/OPT=MAX/MINFT=600	105.0	445.6	5,011.9
/stretch-per/OPT=MAX/MINVT=300	105.0	445.6	5,011.9
/stretch-per/OPT=MAX/MINVT=600	105.0	445.6	5,011.9

Table 6: Average degradation from bound results for the unscaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
FCFS	5,457.2	2,958.5	15,102.7
EASY	4,955.4	2,730.6	14,036.8
Greedy */OPT=AVG	2,527.1	2,472.3	12,487.5
Greedy */OPT=MIN	2,435.0	2,285.6	11,229.9
Greedy/per/OPT=AVG	30.0	10.2	58.1
Greedy/per/OPT=AVG/MINFT=300	26.5	14.4	58.1
Greedy/per/OPT=AVG/MINFT=600	25.6	14.2	57.8
Greedy/per/OPT=AVG/MINVT=300	25.7	14.5	57.8
Greedy/per/OPT=AVG/MINVT=600	25.5	14.2	57.8
Greedy/per/OPT=MIN	30.1	10.2	58.1
Greedy/per/OPT=MIN/MINFT=300	26.0	14.3	58.1
Greedy/per/OPT=MIN/MINFT=600	25.9	14.5	58.0
Greedy/per/OPT=MIN/MINVT=300	25.9	14.5	57.9
Greedy/per/OPT=MIN/MINVT=600	25.9	14.2	58.0
Greedy */per/OPT=AVG	30.5	9.8	65.7
Greedy */per/OPT=AVG/MINFT=300	25.6	14.4	58.7
Greedy */per/OPT=AVG/MINFT=600	25.0	14.3	57.5
Greedy */per/OPT=AVG/MINVT=300	25.3	14.4	57.5
Greedy */per/OPT=AVG/MINVT=600	24.7	14.1	54.1
Greedy */per/OPT=MIN	30.4	9.7	65.7
Greedy */per/OPT=MIN/MINFT=300	25.1	14.3	57.5
Greedy */per/OPT=MIN/MINFT=600	24.9	14.3	57.5
Greedy */per/OPT=MIN/MINVT=300	24.9	14.2	54.1
Greedy */per/OPT=MIN/MINVT=600	24.6	14.3	54.1
GreedyP */OPT=AVG	32.7	146.9	1,230.9
GreedyP */OPT=MIN	37.5	156.0	1,204.9
GreedyP/per/OPT=AVG	20.2	7.2	38.1
GreedyP/per/OPT=AVG/MINFT=300	6.3	4.3	38.1
GreedyP/per/OPT=AVG/MINFT=600	6.1	4.4	38.1
GreedyP/per/OPT=AVG/MINVT=300	6.0	3.9	27.5
GreedyP/per/OPT=AVG/MINVT=600	6.0	4.5	38.1
GreedyP/per/OPT=MIN	20.1	7.3	38.1
GreedyP/per/OPT=MIN/MINFT=300	6.1	3.8	27.5
GreedyP/per/OPT=MIN/MINFT=600	6.1	4.5	38.1
GreedyP/per/OPT=MIN/MINVT=300	5.9	3.8	27.5
GreedyP/per/OPT=MIN/MINVT=600	5.9	4.5	38.1
GreedyP */per/OPT=AVG	20.4	6.8	32.0
GreedyP */per/OPT=AVG/MINFT=300	5.5	2.8	18.0
GreedyP */per/OPT=AVG/MINFT=600	5.1	2.8	18.0
GreedyP */per/OPT=AVG/MINVT=300	4.9	2.4	13.6
GreedyP */per/OPT=AVG/MINVT=600	4.8	2.4	13.6
GreedyP */per/OPT=MIN	20.3	6.8	32.0
GreedyP */per/OPT=MIN/MINFT=300	5.2	2.4	13.7
GreedyP */per/OPT=MIN/MINFT=600	5.0	2.7	18.0
GreedyP */per/OPT=MIN/MINVT=300	4.9	2.7	18.0
GreedyP */per/OPT=MIN/MINVT=600	4.9	2.9	19.2
GreedyPM */OPT=AVG	28.2	104.4	676.2
GreedyPM */OPT=MIN	33.8	154.0	1,321.7

Table 6: (Continued) Average degradation from bound results for the unscaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
GreedyPM/per/OPT=AVG	20.2	7.2	38.1
GreedyPM/per/OPT=AVG/MINFT=300	6.3	3.7	27.5
GreedyPM/per/OPT=AVG/MINFT=600	6.1	4.4	38.1
GreedyPM/per/OPT=AVG/MINVT=300	6.2	4.5	38.1
GreedyPM/per/OPT=AVG/MINVT=600	5.9	4.4	38.1
GreedyPM/per/OPT=MIN	20.2	7.3	38.1
GreedyPM/per/OPT=MIN/MINFT=300	6.1	3.6	27.5
GreedyPM/per/OPT=MIN/MINFT=600	6.0	4.4	38.1
GreedyPM/per/OPT=MIN/MINVT=300	6.0	3.9	27.5
GreedyPM/per/OPT=MIN/MINVT=600	5.9	4.5	38.1
GreedyPM */per/OPT=AVG	20.4	6.8	32.0
GreedyPM */per/OPT=AVG/MINFT=300	5.5	2.6	13.7
GreedyPM */per/OPT=AVG/MINFT=600	5.0	2.5	13.7
GreedyPM */per/OPT=AVG/MINVT=300	4.9	2.5	13.8
GreedyPM */per/OPT=AVG/MINVT=600	4.8	2.4	13.6
GreedyPM */per/OPT=MIN	20.3	6.9	32.0
GreedyPM */per/OPT=MIN/MINFT=300	5.3	2.7	18.0
GreedyPM */per/OPT=MIN/MINFT=600	4.9	2.5	13.7
GreedyPM */per/OPT=MIN/MINVT=300	4.9	2.7	18.0
GreedyPM */per/OPT=MIN/MINVT=600	4.8	2.4	13.6
MCB8 */OPT=AVG	245.1	130.3	634.2
MCB8 */OPT=AVG/MINFT=300	18.0	23.2	206.3
MCB8 */OPT=AVG/MINFT=600	9.8	6.4	43.6
MCB8 */OPT=AVG/MINVT=300	8.6	5.6	43.9
MCB8 */OPT=AVG/MINVT=600	7.7	6.9	44.8
MCB8 */OPT=MIN	233.2	117.1	634.2
MCB8 */OPT=MIN/MINFT=300	16.6	22.8	206.3
MCB8 */OPT=MIN/MINFT=600	9.9	8.1	51.3
MCB8 */OPT=MIN/MINVT=300	9.2	8.0	65.3
MCB8 */OPT=MIN/MINVT=600	6.9	5.4	44.4
MCB8/per/OPT=AVG	134.7	57.1	324.1
MCB8/per/OPT=AVG/MINFT=300	15.2	18.7	173.0
MCB8/per/OPT=AVG/MINFT=600	10.2	8.1	65.7
MCB8/per/OPT=AVG/MINVT=300	9.2	6.8	51.3
MCB8/per/OPT=AVG/MINVT=600	8.2	7.0	53.3
MCB8/per/OPT=MIN	133.7	57.5	323.7
MCB8/per/OPT=MIN/MINFT=300	14.5	18.6	173.0
MCB8/per/OPT=MIN/MINFT=600	10.0	8.1	65.7
MCB8/per/OPT=MIN/MINVT=300	9.0	6.7	51.3
MCB8/per/OPT=MIN/MINVT=600	8.1	6.6	53.3
MCB8 */per/OPT=AVG	252.1	126.3	634.2
MCB8 */per/OPT=AVG/MINFT=300	19.5	35.4	349.2
MCB8 */per/OPT=AVG/MINFT=600	10.7	5.6	37.1
MCB8 */per/OPT=AVG/MINVT=300	8.8	3.5	19.0
MCB8 */per/OPT=AVG/MINVT=600	7.8	3.8	21.4
MCB8 */per/OPT=MIN	250.6	125.0	634.2
MCB8 */per/OPT=MIN/MINFT=300	19.0	35.3	349.2
MCB8 */per/OPT=MIN/MINFT=600	10.6	5.7	37.1

Table 6: (Continued) Average degradation from bound results for the unscaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
MCB8 */per/OPT=MIN/MINVT=300	8.9	3.5	19.0
MCB8 */per/OPT=MIN/MINVT=600	7.8	3.9	21.9
/per/OPT=AVG	43.1	19.7	134.7
/per/OPT=AVG/MINFT=300	43.0	19.7	134.7
/per/OPT=AVG/MINFT=600	43.0	19.7	134.7
/per/OPT=AVG/MINVT=300	43.0	19.8	134.7
/per/OPT=AVG/MINVT=600	43.1	19.7	134.7
/per/OPT=MIN	43.0	19.8	134.7
/per/OPT=MIN/MINFT=300	43.0	19.8	134.7
/per/OPT=MIN/MINFT=600	43.0	19.8	134.7
/per/OPT=MIN/MINVT=300	43.0	19.8	134.7
/per/OPT=MIN/MINVT=600	43.0	19.7	134.7
/stretch-per/OPT=AVG	43.1	19.5	134.7
/stretch-per/OPT=AVG/MINFT=300	43.1	19.5	134.7
/stretch-per/OPT=AVG/MINFT=600	43.1	19.5	134.7
/stretch-per/OPT=AVG/MINVT=300	43.1	19.5	134.7
/stretch-per/OPT=AVG/MINVT=600	43.0	19.6	134.7
/stretch-per/OPT=MAX	43.0	19.6	134.7
/stretch-per/OPT=MAX/MINFT=300	43.0	19.6	134.7
/stretch-per/OPT=MAX/MINFT=600	43.0	19.6	134.7
/stretch-per/OPT=MAX/MINVT=300	43.0	19.6	134.7
/stretch-per/OPT=MAX/MINVT=600	43.0	19.6	134.7

Table 7: Average degradation from bound results for the scaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
FCFS	5,869.3	2,789.1	17,403.3
EASY	5,262.0	2,588.9	14,534.1
Greedy */OPT=AVG	3,326.7	2,561.2	18,310.2
Greedy */OPT=MIN	3,204.3	2,517.5	19,129.2
Greedy/per/OPT=AVG	29.2	14.3	153.2
Greedy/per/OPT=AVG/MINFT=300	27.4	15.5	153.2
Greedy/per/OPT=AVG/MINFT=600	27.3	15.5	152.6
Greedy/per/OPT=AVG/MINVT=300	27.5	15.8	153.2
Greedy/per/OPT=AVG/MINVT=600	27.4	15.9	153.0
Greedy/per/OPT=MIN	29.3	14.3	153.2
Greedy/per/OPT=MIN/MINFT=300	27.4	15.7	153.2
Greedy/per/OPT=MIN/MINFT=600	27.4	15.9	152.6
Greedy/per/OPT=MIN/MINVT=300	27.6	15.9	153.4
Greedy/per/OPT=MIN/MINVT=600	27.0	15.5	152.8
Greedy */per/OPT=AVG	29.2	11.9	101.4
Greedy */per/OPT=AVG/MINFT=300	26.7	13.6	87.1
Greedy */per/OPT=AVG/MINFT=600	25.5	13.2	95.2
Greedy */per/OPT=AVG/MINVT=300	25.6	13.0	81.0
Greedy */per/OPT=AVG/MINVT=600	25.4	13.1	95.2
Greedy */per/OPT=MIN	29.1	12.3	101.4
Greedy */per/OPT=MIN/MINFT=300	26.4	13.2	87.1
Greedy */per/OPT=MIN/MINFT=600	25.5	13.3	103.9
Greedy */per/OPT=MIN/MINVT=300	25.4	13.0	81.0
Greedy */per/OPT=MIN/MINVT=600	25.1	13.0	95.2
GreedyP */OPT=AVG	114.3	617.3	9,490.0
GreedyP */OPT=MIN	115.7	644.0	10,354.2
GreedyP/per/OPT=AVG	18.0	9.7	84.6
GreedyP/per/OPT=AVG/MINFT=300	7.7	7.9	84.6
GreedyP/per/OPT=AVG/MINFT=600	7.5	7.8	84.6
GreedyP/per/OPT=AVG/MINVT=300	7.4	7.8	84.6
GreedyP/per/OPT=AVG/MINVT=600	7.3	8.4	96.8
GreedyP/per/OPT=MIN	17.8	9.6	84.6
GreedyP/per/OPT=MIN/MINFT=300	7.6	7.9	84.6
GreedyP/per/OPT=MIN/MINFT=600	7.3	7.6	84.6
GreedyP/per/OPT=MIN/MINVT=300	7.3	7.7	84.6
GreedyP/per/OPT=MIN/MINVT=600	7.3	8.5	96.8
GreedyP */per/OPT=AVG	18.1	8.6	89.9
GreedyP */per/OPT=AVG/MINFT=300	7.1	5.6	90.2
GreedyP */per/OPT=AVG/MINFT=600	6.7	6.3	103.5
GreedyP */per/OPT=AVG/MINVT=300	6.5	6.7	103.5
GreedyP */per/OPT=AVG/MINVT=600	6.3	6.3	103.5
GreedyP */per/OPT=MIN	17.9	8.6	89.9
GreedyP */per/OPT=MIN/MINFT=300	6.8	6.4	103.5
GreedyP */per/OPT=MIN/MINFT=600	6.3	5.4	90.2
GreedyP */per/OPT=MIN/MINVT=300	6.1	5.4	90.2
GreedyP */per/OPT=MIN/MINVT=600	6.1	6.3	103.5
GreedyPM */OPT=AVG	124.9	658.4	9,404.5
GreedyPM */OPT=MIN	124.0	673.5	9,598.8

Table 7: (Continued) Average degradation from bound results for the scaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
GreedyPM/per/OPT=AVG	18.1	9.9	93.3
GreedyPM/per/OPT=AVG/MINFT=300	7.8	7.5	84.6
GreedyPM/per/OPT=AVG/MINFT=600	7.5	7.5	84.6
GreedyPM/per/OPT=AVG/MINVT=300	7.4	7.5	84.6
GreedyPM/per/OPT=AVG/MINVT=600	7.3	8.0	96.8
GreedyPM/per/OPT=MIN	17.9	9.8	93.0
GreedyPM/per/OPT=MIN/MINFT=300	7.6	7.4	84.6
GreedyPM/per/OPT=MIN/MINFT=600	7.4	7.5	84.6
GreedyPM/per/OPT=MIN/MINVT=300	7.3	7.6	84.6
GreedyPM/per/OPT=MIN/MINVT=600	7.3	8.1	96.8
GreedyPM */per/OPT=AVG	18.2	8.7	89.9
GreedyPM */per/OPT=AVG/MINFT=300	7.1	5.2	80.1
GreedyPM */per/OPT=AVG/MINFT=600	6.8	6.4	103.5
GreedyPM */per/OPT=AVG/MINVT=300	6.4	5.6	90.2
GreedyPM */per/OPT=AVG/MINVT=600	6.5	6.6	103.5
GreedyPM */per/OPT=MIN	17.9	8.6	89.9
GreedyPM */per/OPT=MIN/MINFT=300	6.9	6.5	103.5
GreedyPM */per/OPT=MIN/MINFT=600	6.4	6.3	103.5
GreedyPM */per/OPT=MIN/MINVT=300	6.3	5.6	90.2
GreedyPM */per/OPT=MIN/MINVT=600	6.1	5.4	90.2
MCB8 */OPT=AVG	750.1	1,100.8	6,274.4
MCB8 */OPT=AVG/MINFT=300	121.9	351.2	3,609.6
MCB8 */OPT=AVG/MINFT=600	33.2	95.9	1,509.2
MCB8 */OPT=AVG/MINVT=300	15.3	20.9	270.9
MCB8 */OPT=AVG/MINVT=600	14.5	40.9	1,068.1
MCB8 */OPT=MIN	742.4	1,103.0	6,130.4
MCB8 */OPT=MIN/MINFT=300	117.8	358.4	3,680.3
MCB8 */OPT=MIN/MINFT=600	31.7	78.0	1,216.7
MCB8 */OPT=MIN/MINVT=300	15.7	22.5	270.9
MCB8 */OPT=MIN/MINVT=600	13.2	21.6	270.9
MCB8/per/OPT=AVG	155.8	122.8	913.3
MCB8/per/OPT=AVG/MINFT=300	23.5	26.0	231.5
MCB8/per/OPT=AVG/MINFT=600	15.8	19.2	231.4
MCB8/per/OPT=AVG/MINVT=300	12.1	12.3	127.5
MCB8/per/OPT=AVG/MINVT=600	11.1	12.6	127.5
MCB8/per/OPT=MIN	153.0	118.1	909.5
MCB8/per/OPT=MIN/MINFT=300	22.1	24.0	231.5
MCB8/per/OPT=MIN/MINFT=600	15.2	18.9	231.4
MCB8/per/OPT=MIN/MINVT=300	12.3	14.2	223.0
MCB8/per/OPT=MIN/MINVT=600	11.0	12.6	127.5
MCB8 */per/OPT=AVG	959.5	1,469.0	8,299.4
MCB8 */per/OPT=AVG/MINFT=300	168.2	516.8	5,469.8
MCB8 */per/OPT=AVG/MINFT=600	40.3	155.5	2,941.5
MCB8 */per/OPT=AVG/MINVT=300	14.2	15.7	195.7
MCB8 */per/OPT=AVG/MINVT=600	12.0	14.5	195.7
MCB8 */per/OPT=MIN	956.8	1,486.7	8,398.3
MCB8 */per/OPT=MIN/MINFT=300	161.4	481.8	4,590.5
MCB8 */per/OPT=MIN/MINFT=600	37.9	126.9	2,400.6
MCB8 */per/OPT=MIN/MINVT=300	14.4	17.4	222.2

Table 7: (Continued) Average degradation from bound results for the scaled synthetic traces. All results are for a 5-minute rescheduling penalty.

Algorithm	Degradation from bound		
	avg.	std.	max
MCB8 */per/OPT=MIN/MINVT=600	12.2	15.3	195.7
/per/OPT=AVG	40.4	25.1	238.3
/per/OPT=AVG/MINFT=300	40.4	25.0	238.3
/per/OPT=AVG/MINFT=600	40.4	25.1	238.3
/per/OPT=AVG/MINVT=300	40.4	25.0	238.3
/per/OPT=AVG/MINVT=600	40.4	25.0	238.3
/per/OPT=MIN	40.4	25.1	238.3
/per/OPT=MIN/MINFT=300	40.4	25.1	238.3
/per/OPT=MIN/MINFT=600	40.4	25.1	238.3
/per/OPT=MIN/MINVT=300	40.4	25.0	238.3
/per/OPT=MIN/MINVT=600	40.4	25.0	238.3
/stretch-per/OPT=AVG	40.2	24.8	236.5
/stretch-per/OPT=AVG/MINFT=300	40.2	24.8	236.5
/stretch-per/OPT=AVG/MINFT=600	40.2	24.8	236.5
/stretch-per/OPT=AVG/MINVT=300	40.2	24.8	236.5
/stretch-per/OPT=AVG/MINVT=600	40.2	24.8	236.5
/stretch-per/OPT=MAX	40.2	24.8	236.9
/stretch-per/OPT=MAX/MINFT=300	40.2	24.8	236.9
/stretch-per/OPT=MAX/MINFT=600	40.2	24.8	236.9
/stretch-per/OPT=MAX/MINVT=300	40.2	24.8	236.9
/stretch-per/OPT=MAX/MINVT=600	40.2	24.8	236.9

Table 8: Preemption and migration bandwidth consumption for DFRS algorithms. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Bandwidth consumption (GB / sec)			
	pmtn		mig	
	avg.	max	avg.	max
Greedy */OPT=AVG	0.00	0.00	0.00	0.00
Greedy */OPT=MIN	0.00	0.00	0.00	0.00
Greedy/per/OPT=AVG	0.44	1.02	0.21	0.63
Greedy/per/OPT=AVG/MINFT=300	0.44	1.04	0.20	0.62
Greedy/per/OPT=AVG/MINFT=600	0.44	1.03	0.20	0.62
Greedy/per/OPT=AVG/MINVT=300	0.44	1.03	0.20	0.63
Greedy/per/OPT=AVG/MINVT=600	0.44	1.04	0.19	0.60
Greedy/per/OPT=MIN	0.48	1.08	0.21	0.60
Greedy/per/OPT=MIN/MINFT=300	0.47	1.08	0.20	0.59
Greedy/per/OPT=MIN/MINFT=600	0.47	1.08	0.19	0.58
Greedy/per/OPT=MIN/MINVT=300	0.47	1.06	0.19	0.57
Greedy/per/OPT=MIN/MINVT=600	0.47	1.07	0.18	0.58
Greedy */per/OPT=AVG	0.45	1.28	0.28	0.69
Greedy */per/OPT=AVG/MINFT=300	0.45	1.27	0.27	0.66
Greedy */per/OPT=AVG/MINFT=600	0.44	1.26	0.26	0.67
Greedy */per/OPT=AVG/MINVT=300	0.44	1.26	0.26	0.65
Greedy */per/OPT=AVG/MINVT=600	0.44	1.26	0.25	0.65
Greedy */per/OPT=MIN	0.50	1.29	0.27	0.66
Greedy */per/OPT=MIN/MINFT=300	0.50	1.29	0.26	0.65
Greedy */per/OPT=MIN/MINFT=600	0.50	1.29	0.26	0.63
Greedy */per/OPT=MIN/MINVT=300	0.50	1.27	0.26	0.63
Greedy */per/OPT=MIN/MINVT=600	0.49	1.27	0.24	0.62
GreedyP */OPT=AVG	0.06	0.17	0.00	0.00
GreedyP */OPT=MIN	0.06	0.17	0.00	0.00
GreedyP/per/OPT=AVG	0.46	1.05	0.20	0.64
GreedyP/per/OPT=AVG/MINFT=300	0.46	1.07	0.19	0.61
GreedyP/per/OPT=AVG/MINFT=600	0.46	1.06	0.19	0.60
GreedyP/per/OPT=AVG/MINVT=300	0.46	1.07	0.19	0.62
GreedyP/per/OPT=AVG/MINVT=600	0.46	1.05	0.18	0.60
GreedyP/per/OPT=MIN	0.50	1.11	0.20	0.60
GreedyP/per/OPT=MIN/MINFT=300	0.49	1.11	0.19	0.58
GreedyP/per/OPT=MIN/MINFT=600	0.49	1.10	0.18	0.58
GreedyP/per/OPT=MIN/MINVT=300	0.49	1.10	0.18	0.57
GreedyP/per/OPT=MIN/MINVT=600	0.49	1.11	0.18	0.57
GreedyP */per/OPT=AVG	0.53	1.36	0.28	0.68
GreedyP */per/OPT=AVG/MINFT=300	0.52	1.35	0.27	0.69
GreedyP */per/OPT=AVG/MINFT=600	0.52	1.36	0.26	0.69
GreedyP */per/OPT=AVG/MINVT=300	0.52	1.35	0.26	0.66
GreedyP */per/OPT=AVG/MINVT=600	0.51	1.35	0.25	0.65
GreedyP */per/OPT=MIN	0.58	1.37	0.28	0.65
GreedyP */per/OPT=MIN/MINFT=300	0.57	1.38	0.26	0.65
GreedyP */per/OPT=MIN/MINFT=600	0.57	1.38	0.26	0.65
GreedyP */per/OPT=MIN/MINVT=300	0.57	1.37	0.25	0.62
GreedyP */per/OPT=MIN/MINVT=600	0.56	1.36	0.24	0.63
GreedyPM */OPT=AVG	0.03	0.08	0.02	0.05

Table 8: (Continued) Preemption and migration bandwidth consumption for DFRS algorithms. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Bandwidth consumption (GB / sec)			
	pmtn		mig	
	avg.	max	avg.	max
GreedyPM */OPT=MIN	0.03	0.07	0.02	0.05
GreedyPM/per/OPT=AVG	0.46	1.05	0.21	0.64
GreedyPM/per/OPT=AVG/MINFT=300	0.46	1.05	0.20	0.64
GreedyPM/per/OPT=AVG/MINFT=600	0.45	1.04	0.20	0.61
GreedyPM/per/OPT=AVG/MINVT=300	0.45	1.06	0.20	0.62
GreedyPM/per/OPT=AVG/MINVT=600	0.45	1.06	0.19	0.60
GreedyPM/per/OPT=MIN	0.49	1.10	0.21	0.60
GreedyPM/per/OPT=MIN/MINFT=300	0.49	1.10	0.20	0.61
GreedyPM/per/OPT=MIN/MINFT=600	0.49	1.10	0.19	0.58
GreedyPM/per/OPT=MIN/MINVT=300	0.49	1.10	0.19	0.57
GreedyPM/per/OPT=MIN/MINVT=600	0.49	1.10	0.18	0.57
GreedyPM */per/OPT=AVG	0.51	1.33	0.29	0.68
GreedyPM */per/OPT=AVG/MINFT=300	0.50	1.33	0.28	0.69
GreedyPM */per/OPT=AVG/MINFT=600	0.50	1.35	0.27	0.68
GreedyPM */per/OPT=AVG/MINVT=300	0.49	1.33	0.27	0.67
GreedyPM */per/OPT=AVG/MINVT=600	0.49	1.34	0.26	0.65
GreedyPM */per/OPT=MIN	0.56	1.37	0.29	0.66
GreedyPM */per/OPT=MIN/MINFT=300	0.55	1.36	0.27	0.66
GreedyPM */per/OPT=MIN/MINFT=600	0.55	1.36	0.27	0.65
GreedyPM */per/OPT=MIN/MINVT=300	0.55	1.36	0.27	0.64
GreedyPM */per/OPT=MIN/MINVT=600	0.54	1.34	0.26	0.62
MCB8 */OPT=AVG	0.38	1.15	1.16	2.35
MCB8 */OPT=AVG/MINFT=300	0.18	0.99	0.86	2.98
MCB8 */OPT=AVG/MINFT=600	0.14	0.67	0.73	2.60
MCB8 */OPT=AVG/MINVT=300	0.13	0.48	0.61	2.09
MCB8 */OPT=AVG/MINVT=600	0.12	0.39	0.53	1.49
MCB8 */OPT=MIN	0.42	1.26	1.17	2.36
MCB8 */OPT=MIN/MINFT=300	0.20	0.98	0.86	2.89
MCB8 */OPT=MIN/MINFT=600	0.15	0.78	0.75	2.72
MCB8 */OPT=MIN/MINVT=300	0.13	0.72	0.63	2.12
MCB8 */OPT=MIN/MINVT=600	0.13	0.37	0.53	1.51
MCB8/per/OPT=AVG	0.52	1.07	0.69	2.92
MCB8/per/OPT=AVG/MINFT=300	0.50	1.05	0.55	2.14
MCB8/per/OPT=AVG/MINFT=600	0.49	1.04	0.51	1.66
MCB8/per/OPT=AVG/MINVT=300	0.49	1.07	0.47	1.32
MCB8/per/OPT=AVG/MINVT=600	0.49	1.06	0.43	1.18
MCB8/per/OPT=MIN	0.56	1.10	0.69	3.03
MCB8/per/OPT=MIN/MINFT=300	0.54	1.10	0.55	2.04
MCB8/per/OPT=MIN/MINFT=600	0.53	1.09	0.51	1.68
MCB8/per/OPT=MIN/MINVT=300	0.53	1.11	0.47	1.40
MCB8/per/OPT=MIN/MINVT=600	0.53	1.12	0.43	1.12
MCB8 */per/OPT=AVG	0.67	1.08	1.21	2.54
MCB8 */per/OPT=AVG/MINFT=300	0.54	1.05	0.89	3.12
MCB8 */per/OPT=AVG/MINFT=600	0.51	1.03	0.77	2.69
MCB8 */per/OPT=AVG/MINVT=300	0.50	1.07	0.65	2.08
MCB8 */per/OPT=AVG/MINVT=600	0.50	1.09	0.57	1.53

Table 8: (Continued) Preemption and migration bandwidth consumption for DFRS algorithms. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Bandwidth consumption (GB / sec)			
	pmtn		mig	
	avg.	max	avg.	max
MCB8 */per/OPT=MIN	0.72	1.15	1.21	2.68
MCB8 */per/OPT=MIN/MINFT=300	0.58	1.12	0.89	3.02
MCB8 */per/OPT=MIN/MINFT=600	0.55	1.10	0.77	2.76
MCB8 */per/OPT=MIN/MINVT=300	0.54	1.11	0.65	2.16
MCB8 */per/OPT=MIN/MINVT=600	0.54	1.11	0.56	1.53
/per/OPT=AVG	0.45	1.02	0.21	0.64
/per/OPT=AVG/MINFT=300	0.45	1.03	0.21	0.64
/per/OPT=AVG/MINFT=600	0.45	1.04	0.21	0.64
/per/OPT=AVG/MINVT=300	0.45	1.02	0.21	0.63
/per/OPT=AVG/MINVT=600	0.45	1.03	0.20	0.62
/per/OPT=MIN	0.49	1.07	0.21	0.62
/per/OPT=MIN/MINFT=300	0.49	1.07	0.21	0.62
/per/OPT=MIN/MINFT=600	0.49	1.07	0.21	0.62
/per/OPT=MIN/MINVT=300	0.49	1.08	0.20	0.60
/per/OPT=MIN/MINVT=600	0.49	1.08	0.19	0.58
/stretch-per/OPT=AVG	0.28	0.66	0.39	0.79
/stretch-per/OPT=AVG/MINFT=300	0.28	0.66	0.39	0.78
/stretch-per/OPT=AVG/MINFT=600	0.28	0.66	0.39	0.78
/stretch-per/OPT=AVG/MINVT=300	0.28	0.68	0.38	0.79
/stretch-per/OPT=AVG/MINVT=600	0.28	0.68	0.37	0.78
/stretch-per/OPT=MAX	0.28	0.65	0.39	0.81
/stretch-per/OPT=MAX/MINFT=300	0.28	0.65	0.39	0.81
/stretch-per/OPT=MAX/MINFT=600	0.28	0.65	0.39	0.81
/stretch-per/OPT=MAX/MINVT=300	0.28	0.65	0.38	0.81
/stretch-per/OPT=MAX/MINVT=600	0.28	0.64	0.37	0.78

Table 9: Preemption and migration frequency in terms of number of preemption and migration occurrences per hour. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / hour			
	pmtn		mig	
	avg.	max	avg.	max
Greedy */OPT=AVG	0.00	0.00	0.00	0.00
Greedy */OPT=MIN	0.00	0.00	0.00	0.00
Greedy/per/OPT=AVG	30.12	75.96	38.81	124.56
Greedy/per/OPT=AVG/MINFT=300	29.82	75.24	36.41	110.88
Greedy/per/OPT=AVG/MINFT=600	29.63	75.96	35.46	112.32
Greedy/per/OPT=AVG/MINVT=300	29.69	74.88	35.46	118.08
Greedy/per/OPT=AVG/MINVT=600	29.55	74.52	33.91	107.64
Greedy/per/OPT=MIN	32.58	83.52	38.79	110.52
Greedy/per/OPT=MIN/MINFT=300	32.28	82.80	36.23	107.28
Greedy/per/OPT=MIN/MINFT=600	32.18	82.80	35.15	106.56
Greedy/per/OPT=MIN/MINVT=300	32.25	82.80	35.11	103.68
Greedy/per/OPT=MIN/MINVT=600	32.04	83.16	33.60	103.68
Greedy */per/OPT=AVG	26.07	73.08	55.45	126.00
Greedy */per/OPT=AVG/MINFT=300	25.68	71.64	53.16	119.16
Greedy */per/OPT=AVG/MINFT=600	25.39	71.28	51.96	117.36
Greedy */per/OPT=AVG/MINVT=300	25.23	71.28	51.74	123.12
Greedy */per/OPT=AVG/MINVT=600	24.86	70.56	50.07	120.24
Greedy */per/OPT=MIN	29.27	84.96	58.06	124.56
Greedy */per/OPT=MIN/MINFT=300	28.74	83.16	55.46	123.84
Greedy */per/OPT=MIN/MINFT=600	28.58	83.88	54.32	123.12
Greedy */per/OPT=MIN/MINVT=300	28.50	83.52	53.86	120.96
Greedy */per/OPT=MIN/MINVT=600	28.08	83.52	51.97	117.36
GreedyP */OPT=AVG	5.78	20.52	0.00	0.00
GreedyP */OPT=MIN	5.67	18.00	0.00	0.00
GreedyP/per/OPT=AVG	30.86	76.68	37.70	111.96
GreedyP/per/OPT=AVG/MINFT=300	30.46	74.88	35.37	108.72
GreedyP/per/OPT=AVG/MINFT=600	30.31	75.24	34.51	110.88
GreedyP/per/OPT=AVG/MINVT=300	30.39	77.04	34.63	106.56
GreedyP/per/OPT=AVG/MINVT=600	30.17	77.04	33.08	103.68
GreedyP/per/OPT=MIN	33.34	85.32	37.52	107.64
GreedyP/per/OPT=MIN/MINFT=300	32.95	83.88	35.05	104.40
GreedyP/per/OPT=MIN/MINFT=600	32.79	84.60	34.07	103.68
GreedyP/per/OPT=MIN/MINVT=300	32.88	84.60	34.17	105.84
GreedyP/per/OPT=MIN/MINVT=600	32.70	83.52	32.74	101.52
GreedyP */per/OPT=AVG	37.08	83.52	56.42	123.12
GreedyP */per/OPT=AVG/MINFT=300	35.90	84.96	53.48	123.12
GreedyP */per/OPT=AVG/MINFT=600	35.58	84.60	52.38	120.60
GreedyP */per/OPT=AVG/MINVT=300	35.33	84.24	52.26	117.00
GreedyP */per/OPT=AVG/MINVT=600	34.70	82.80	50.41	114.84
GreedyP */per/OPT=MIN	39.67	97.92	58.56	127.08
GreedyP */per/OPT=MIN/MINFT=300	38.67	98.28	55.64	124.20
GreedyP */per/OPT=MIN/MINFT=600	38.39	96.48	54.46	122.40
GreedyP */per/OPT=MIN/MINVT=300	38.27	97.56	53.82	119.52
GreedyP */per/OPT=MIN/MINVT=600	37.70	97.20	52.14	119.16
GreedyPM */OPT=AVG	2.31	9.72	3.75	14.04
GreedyPM */OPT=MIN	2.25	10.08	3.69	13.32

Table 9: (Continued) Preemption and migration frequency in terms of number of preemption and migration occurrences per hour. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / hour			
	pmtn		mig	
	avg.	max	avg.	max
GreedyPM/per/OPT=AVG	30.36	77.04	39.63	114.84
GreedyPM/per/OPT=AVG/MINFT=300	29.96	75.24	37.04	114.12
GreedyPM/per/OPT=AVG/MINFT=600	29.79	77.40	36.13	109.80
GreedyPM/per/OPT=AVG/MINVT=300	29.90	75.60	36.03	113.40
GreedyPM/per/OPT=AVG/MINVT=600	29.70	77.04	34.83	116.28
GreedyPM/per/OPT=MIN	32.93	84.24	39.21	112.32
GreedyPM/per/OPT=MIN/MINFT=300	32.46	84.60	36.78	108.36
GreedyPM/per/OPT=MIN/MINFT=600	32.34	83.88	35.87	108.72
GreedyPM/per/OPT=MIN/MINVT=300	32.44	84.24	35.78	108.72
GreedyPM/per/OPT=MIN/MINVT=600	32.24	83.16	34.31	106.56
GreedyPM */per/OPT=AVG	32.78	80.28	60.90	129.60
GreedyPM */per/OPT=AVG/MINFT=300	31.93	81.00	57.83	123.48
GreedyPM */per/OPT=AVG/MINFT=600	31.57	81.00	56.72	119.88
GreedyPM */per/OPT=AVG/MINVT=300	31.19	78.84	56.59	121.32
GreedyPM */per/OPT=AVG/MINVT=600	30.60	79.56	54.88	120.96
GreedyPM */per/OPT=MIN	35.78	96.48	62.95	135.72
GreedyPM */per/OPT=MIN/MINFT=300	34.88	95.04	59.84	132.48
GreedyPM */per/OPT=MIN/MINFT=600	34.45	92.88	58.68	130.32
GreedyPM */per/OPT=MIN/MINVT=300	34.25	94.32	58.31	128.52
GreedyPM */per/OPT=MIN/MINVT=600	33.80	94.32	56.45	127.08
MCB8 */OPT=AVG	56.54	214.56	470.14	998.28
MCB8 */OPT=AVG/MINFT=300	20.23	162.72	327.56	1,313.64
MCB8 */OPT=AVG/MINFT=600	12.86	90.36	279.56	1,201.68
MCB8 */OPT=AVG/MINVT=300	10.04	51.84	238.86	1,101.60
MCB8 */OPT=AVG/MINVT=600	9.27	37.80	206.62	822.60
MCB8 */OPT=MIN	61.66	230.40	490.48	1,005.48
MCB8 */OPT=MIN/MINFT=300	21.60	174.96	337.86	1,343.16
MCB8 */OPT=MIN/MINFT=600	13.24	102.96	291.29	1,306.80
MCB8 */OPT=MIN/MINVT=300	10.25	78.48	247.80	1,083.24
MCB8 */OPT=MIN/MINVT=600	9.51	38.16	212.16	825.48
MCB8/per/OPT=AVG	38.66	83.52	235.57	1,181.88
MCB8/per/OPT=AVG/MINFT=300	35.19	80.28	164.35	798.48
MCB8/per/OPT=AVG/MINFT=600	34.72	79.56	149.35	709.92
MCB8/per/OPT=AVG/MINVT=300	34.57	80.64	139.75	600.84
MCB8/per/OPT=AVG/MINVT=600	34.15	78.48	124.47	480.24
MCB8/per/OPT=MIN	41.59	86.76	243.13	1,269.00
MCB8/per/OPT=MIN/MINFT=300	38.09	86.04	167.67	826.56
MCB8/per/OPT=MIN/MINFT=600	37.47	84.96	151.97	755.64
MCB8/per/OPT=MIN/MINVT=300	37.32	86.40	142.42	619.92
MCB8/per/OPT=MIN/MINVT=600	36.93	86.76	126.13	490.68
MCB8 */per/OPT=AVG	70.98	163.44	461.18	1,045.44
MCB8 */per/OPT=AVG/MINFT=300	44.52	128.52	314.64	1,351.80
MCB8 */per/OPT=AVG/MINFT=600	38.07	83.52	263.46	1,245.60
MCB8 */per/OPT=AVG/MINVT=300	35.99	80.64	222.61	1,057.32
MCB8 */per/OPT=AVG/MINVT=600	35.09	77.04	191.91	795.24
MCB8 */per/OPT=MIN	77.04	170.28	479.31	1,109.52
MCB8 */per/OPT=MIN/MINFT=300	47.63	135.00	325.09	1,370.16

Table 9: (Continued) Preemption and migration frequency in terms of number of preemption and migration occurrences per hour. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / hour			
	pmtn		mig	
	avg.	max	avg.	max
MCB8 */per/OPT=MIN/MINFT=600	41.08	93.60	271.59	1,337.76
MCB8 */per/OPT=MIN/MINVT=300	38.88	86.40	227.80	1,129.32
MCB8 */per/OPT=MIN/MINVT=600	37.94	85.32	194.57	836.28
/per/OPT=AVG	31.29	76.32	38.76	113.76
/per/OPT=AVG/MINFT=300	31.17	75.96	39.00	114.84
/per/OPT=AVG/MINFT=600	31.23	75.96	38.96	113.40
/per/OPT=AVG/MINVT=300	31.16	75.24	37.86	111.24
/per/OPT=AVG/MINVT=600	31.04	75.24	36.62	111.60
/per/OPT=MIN	33.83	84.24	38.69	111.24
/per/OPT=MIN/MINFT=300	33.83	84.24	38.69	111.24
/per/OPT=MIN/MINFT=600	33.83	84.24	38.69	111.24
/per/OPT=MIN/MINVT=300	33.88	83.52	37.62	107.64
/per/OPT=MIN/MINVT=600	33.76	84.60	36.21	104.04
/stretch-per/OPT=AVG	20.64	43.20	62.89	140.04
/stretch-per/OPT=AVG/MINFT=300	20.65	42.48	63.03	140.40
/stretch-per/OPT=AVG/MINFT=600	20.62	43.56	62.88	138.96
/stretch-per/OPT=AVG/MINVT=300	20.62	43.20	61.58	136.80
/stretch-per/OPT=AVG/MINVT=600	20.60	43.56	59.78	132.48
/stretch-per/OPT=MAX	20.41	45.36	67.26	159.48
/stretch-per/OPT=MAX/MINFT=300	20.41	45.36	67.26	159.48
/stretch-per/OPT=MAX/MINFT=600	20.41	45.36	67.26	159.48
/stretch-per/OPT=MAX/MINVT=300	20.35	46.44	65.60	158.40
/stretch-per/OPT=MAX/MINVT=600	20.25	46.80	63.87	153.36

Table 10: Preemption and migration frequency in terms of number of preemption and migration occurrences per job. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / job			
	pmtn		mig	
	avg.	max	avg.	max
Greedy */OPT=AVG	0.00	0.00	0.00	0.00
Greedy */OPT=MIN	0.00	0.00	0.00	0.00
Greedy/per/OPT=AVG	5.03	21.58	4.79	18.26
Greedy/per/OPT=AVG/MINFT=300	4.98	21.52	4.52	15.80
Greedy/per/OPT=AVG/MINFT=600	4.94	20.66	4.41	16.26
Greedy/per/OPT=AVG/MINVT=300	4.95	21.03	4.40	17.08
Greedy/per/OPT=AVG/MINVT=600	4.91	21.01	4.22	15.65
Greedy/per/OPT=MIN	5.41	21.76	4.81	16.17
Greedy/per/OPT=MIN/MINFT=300	5.36	21.55	4.52	15.40
Greedy/per/OPT=MIN/MINFT=600	5.33	21.31	4.39	15.27
Greedy/per/OPT=MIN/MINVT=300	5.34	21.83	4.37	15.21
Greedy/per/OPT=MIN/MINVT=600	5.29	21.94	4.20	14.83
Greedy */per/OPT=AVG	3.95	20.26	6.59	17.29
Greedy */per/OPT=AVG/MINFT=300	3.89	19.76	6.36	16.62
Greedy */per/OPT=AVG/MINFT=600	3.85	19.57	6.23	16.52
Greedy */per/OPT=AVG/MINVT=300	3.81	19.60	6.18	16.42
Greedy */per/OPT=AVG/MINVT=600	3.75	19.24	5.98	16.01
Greedy */per/OPT=MIN	4.49	22.55	6.94	17.65
Greedy */per/OPT=MIN/MINFT=300	4.41	22.07	6.66	17.31
Greedy */per/OPT=MIN/MINFT=600	4.38	21.89	6.55	17.38
Greedy */per/OPT=MIN/MINVT=300	4.37	21.71	6.48	17.26
Greedy */per/OPT=MIN/MINVT=600	4.29	21.80	6.25	16.53
GreedyP */OPT=AVG	0.58	2.09	0.00	0.00
GreedyP */OPT=MIN	0.57	2.04	0.00	0.00
GreedyP/per/OPT=AVG	5.16	21.11	4.67	16.20
GreedyP/per/OPT=AVG/MINFT=300	5.09	21.05	4.41	15.45
GreedyP/per/OPT=AVG/MINFT=600	5.05	20.97	4.32	15.12
GreedyP/per/OPT=AVG/MINVT=300	5.06	20.52	4.33	15.29
GreedyP/per/OPT=AVG/MINVT=600	5.02	20.36	4.14	14.68
GreedyP/per/OPT=MIN	5.54	22.20	4.67	15.76
GreedyP/per/OPT=MIN/MINFT=300	5.47	21.45	4.39	15.09
GreedyP/per/OPT=MIN/MINFT=600	5.45	21.41	4.28	15.30
GreedyP/per/OPT=MIN/MINVT=300	5.46	21.94	4.29	15.04
GreedyP/per/OPT=MIN/MINVT=600	5.41	21.37	4.11	14.29
GreedyP */per/OPT=AVG	5.37	22.68	6.78	17.07
GreedyP */per/OPT=AVG/MINFT=300	5.21	22.27	6.47	17.30
GreedyP */per/OPT=AVG/MINFT=600	5.17	21.34	6.35	17.24
GreedyP */per/OPT=AVG/MINVT=300	5.12	21.58	6.32	16.78
GreedyP */per/OPT=AVG/MINVT=600	5.03	21.52	6.09	16.26
GreedyP */per/OPT=MIN	5.87	24.87	7.09	17.91
GreedyP */per/OPT=MIN/MINFT=300	5.73	23.92	6.78	17.45
GreedyP */per/OPT=MIN/MINFT=600	5.69	24.22	6.65	17.17
GreedyP */per/OPT=MIN/MINVT=300	5.66	24.30	6.54	17.04
GreedyP */per/OPT=MIN/MINVT=600	5.56	23.88	6.34	16.48
GreedyPM */OPT=AVG	0.24	1.11	0.37	1.18
GreedyPM */OPT=MIN	0.23	1.19	0.36	1.22

Table 10: (Continued) Preemption and migration frequency in terms of number of preemption and migration occurrences per job. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / job			
	pmtn		mig	
	avg.	max	avg.	max
GreedyPM/per/OPT=AVG	5.11	20.79	4.88	16.67
GreedyPM/per/OPT=AVG/MINFT=300	5.03	20.61	4.60	15.78
GreedyPM/per/OPT=AVG/MINFT=600	5.01	20.41	4.50	15.52
GreedyPM/per/OPT=AVG/MINVT=300	5.02	20.99	4.48	15.41
GreedyPM/per/OPT=AVG/MINVT=600	4.98	21.06	4.33	16.83
GreedyPM/per/OPT=MIN	5.52	21.65	4.85	16.03
GreedyPM/per/OPT=MIN/MINFT=300	5.43	21.97	4.58	15.71
GreedyPM/per/OPT=MIN/MINFT=600	5.41	21.39	4.49	15.59
GreedyPM/per/OPT=MIN/MINVT=300	5.42	21.67	4.45	15.15
GreedyPM/per/OPT=MIN/MINVT=600	5.37	22.19	4.28	15.07
GreedyPM */per/OPT=AVG	4.88	22.02	7.30	18.27
GreedyPM */per/OPT=AVG/MINFT=300	4.76	21.49	6.98	17.71
GreedyPM */per/OPT=AVG/MINFT=600	4.70	21.30	6.85	17.02
GreedyPM */per/OPT=AVG/MINVT=300	4.65	21.03	6.81	17.11
GreedyPM */per/OPT=AVG/MINVT=600	4.55	20.70	6.61	17.24
GreedyPM */per/OPT=MIN	5.42	24.00	7.59	18.32
GreedyPM */per/OPT=MIN/MINFT=300	5.29	23.42	7.27	18.23
GreedyPM */per/OPT=MIN/MINFT=600	5.23	23.76	7.14	17.80
GreedyPM */per/OPT=MIN/MINVT=300	5.19	23.12	7.07	17.32
GreedyPM */per/OPT=MIN/MINVT=600	5.11	23.23	6.84	16.94
MCB8 */OPT=AVG	6.10	17.36	55.01	86.16
MCB8 */OPT=AVG/MINFT=300	2.02	14.26	31.45	80.96
MCB8 */OPT=AVG/MINFT=600	1.31	9.68	25.24	65.40
MCB8 */OPT=AVG/MINVT=300	1.03	2.88	20.89	42.69
MCB8 */OPT=AVG/MINVT=600	0.97	3.42	18.17	41.67
MCB8 */OPT=MIN	6.67	18.83	57.46	90.29
MCB8 */OPT=MIN/MINFT=300	2.18	15.58	32.59	89.17
MCB8 */OPT=MIN/MINFT=600	1.37	8.67	26.28	68.39
MCB8 */OPT=MIN/MINVT=300	1.06	3.08	21.73	46.10
MCB8 */OPT=MIN/MINVT=600	1.00	2.89	18.68	41.15
MCB8/per/OPT=AVG	6.34	25.03	25.02	41.57
MCB8/per/OPT=AVG/MINFT=300	6.04	24.52	18.31	36.68
MCB8/per/OPT=AVG/MINFT=600	5.97	24.57	16.87	35.67
MCB8/per/OPT=AVG/MINVT=300	5.94	25.07	15.76	33.88
MCB8/per/OPT=AVG/MINVT=600	5.85	24.90	14.18	33.19
MCB8/per/OPT=MIN	6.83	26.38	25.74	43.24
MCB8/per/OPT=MIN/MINFT=300	6.51	26.03	18.75	38.89
MCB8/per/OPT=MIN/MINFT=600	6.42	26.10	17.23	36.68
MCB8/per/OPT=MIN/MINVT=300	6.39	26.37	16.13	36.79
MCB8/per/OPT=MIN/MINVT=600	6.30	26.13	14.45	34.48
MCB8 */per/OPT=AVG	11.97	26.78	70.57	106.14
MCB8 */per/OPT=AVG/MINFT=300	7.54	26.28	39.94	105.55
MCB8 */per/OPT=AVG/MINFT=600	6.59	24.99	31.32	78.30
MCB8 */per/OPT=AVG/MINVT=300	6.24	25.69	25.53	53.96
MCB8 */per/OPT=AVG/MINVT=600	6.08	25.58	22.24	50.41
MCB8 */per/OPT=MIN	13.01	28.93	73.61	117.15
MCB8 */per/OPT=MIN/MINFT=300	8.11	27.86	41.44	112.18

Table 10: (Continued) Preemption and migration frequency in terms of number of preemption and migration occurrences per job. Average and maximum values over scaled synthetic traces with load ≥ 0.7.

Algorithm	Occurrences / job			
	pmtn		mig	
	avg.	max	avg.	max
MCB8 */per/OPT=MIN/MINFT=600	7.13	27.14	32.52	81.60
MCB8 */per/OPT=MIN/MINVT=300	6.72	27.27	26.33	54.97
MCB8 */per/OPT=MIN/MINVT=600	6.57	26.52	22.55	50.29
/per/OPT=AVG	5.25	22.34	4.89	16.87
/per/OPT=AVG/MINFT=300	5.24	22.85	4.92	17.02
/per/OPT=AVG/MINFT=600	5.24	22.90	4.92	16.88
/per/OPT=AVG/MINVT=300	5.23	22.16	4.78	16.52
/per/OPT=AVG/MINVT=600	5.20	22.95	4.63	16.52
/per/OPT=MIN	5.65	23.23	4.90	16.58
/per/OPT=MIN/MINFT=300	5.65	23.23	4.90	16.58
/per/OPT=MIN/MINFT=600	5.65	23.23	4.90	16.58
/per/OPT=MIN/MINVT=300	5.65	23.25	4.77	16.00
/per/OPT=MIN/MINVT=600	5.63	22.78	4.59	15.51
/stretch-per/OPT=AVG	3.79	16.03	9.58	23.98
/stretch-per/OPT=AVG/MINFT=300	3.79	15.86	9.60	24.02
/stretch-per/OPT=AVG/MINFT=600	3.78	16.13	9.58	23.79
/stretch-per/OPT=AVG/MINVT=300	3.77	16.24	9.36	23.27
/stretch-per/OPT=AVG/MINVT=600	3.77	16.00	9.11	22.63
/stretch-per/OPT=MAX	3.79	16.71	10.41	26.96
/stretch-per/OPT=MAX/MINFT=300	3.79	16.71	10.41	26.96
/stretch-per/OPT=MAX/MINFT=600	3.79	16.71	10.41	26.96
/stretch-per/OPT=MAX/MINVT=300	3.78	17.13	10.14	26.75
/stretch-per/OPT=MAX/MINVT=600	3.76	17.52	9.87	25.78

B Additional Graphs

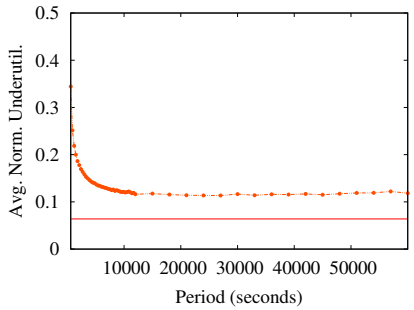

Figure 5: Average normalized underutilization vs. period for EASY (solid) and GreedyPM */per/OPT=MIN/MINVT=600 (dots) on Real-world traces, to 60,000 seconds

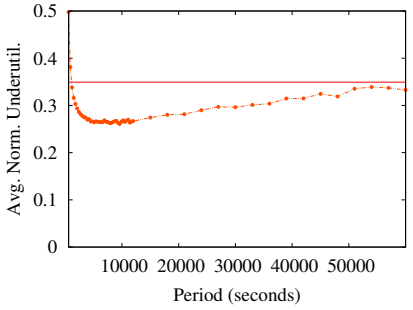

Figure 6: Average normalized underutilization vs. period for EASY (solid) and GreedyPM */per/OPT=MIN/MINVT=600 (dots) on Unscaled synthetic traces, to 60,000 seconds

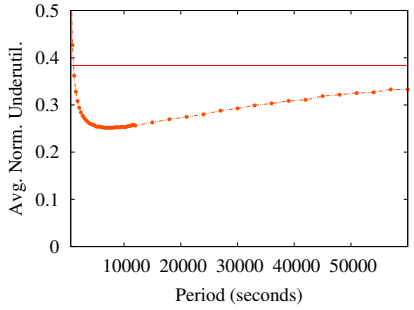

Figure 7: Average normalized underutilization vs. period for EASY (solid) and GreedyPM */per/OPT=MIN/MINVT=600 (dots) on Scaled synthetic traces, to 60,000 seconds

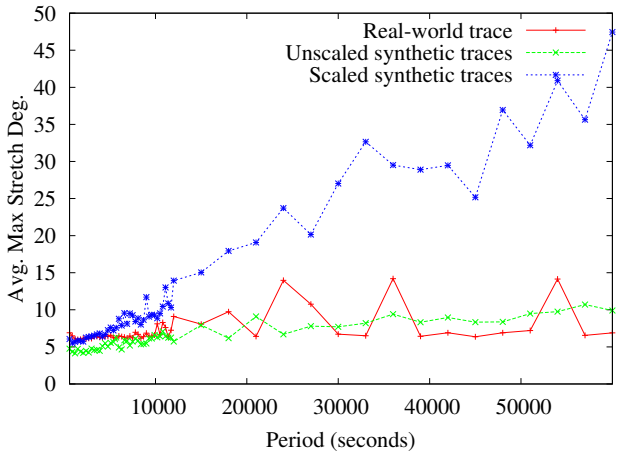

Figure 8: Maximum stretch degradation from bound vs. scheduling period for GreedyPM */per/OPT=MIN/MINVT=600 for all three trace sets, to 60,000 seconds

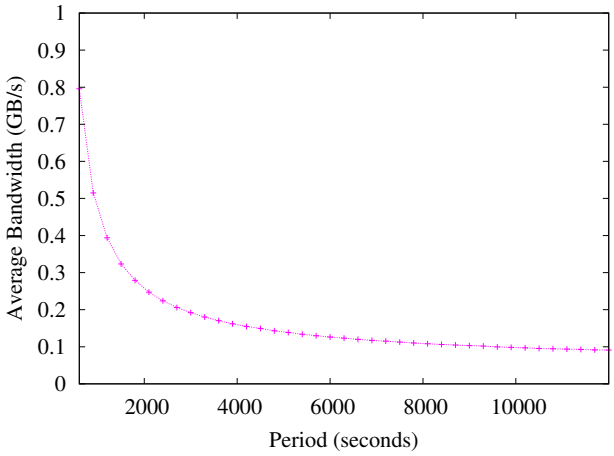

Figure 9: Bandwidth consumption vs. period for GreedyPM */per/OPT=MIN/MINVT=600 for the scaled synthetic traces with load values ≥ 0.7, to 12,000 seconds

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