Stepwise Development of Distributed Algorithms (Research Abstract)

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Abstract. We study distributed algorithms using the "*correct-by-construction* process", which involves the Event B method and the refinement process. These algorithms integrate non-functional requirements like probabilistic aspects. Therefore, it appears that the *correct-by-construction* process should integrate probabilistic arguments to be considered as a development and verification framework for distributed algorithms. We present here preliminary elements of a global approach on how to take into account probabilistic arguments during the development of distributed algorithms, and we illustrate this approach with the study of distributed coloring algorithms.

1 Introduction

Overview - Verifying distributed algorithms is non trivial. A promising methodology is based on the Event B language and the refinement process ([13]): it consists in redeveloping the algorithms by targeting a collection of required properties, using the refinement of models which is preserving the properties of an abstract one in concrete ones. However, this approach called "*correct-by-construction* process" does not handle probabilistic arguments. Therefore, we propose in this paper a way of handling probabilities and integrating them into the process, and we illustrate our approach through the study of vertex coloring algorithms, which have strong features related to probabilistic statements.

Related works - The *correct-by-construction* approach has already been followed by Abrial et al during their redevelopment of the tree identification protocol IEEE 1394 ([2]), leading to an algorithm which does not integrate *probabilistic* arguments. In this paper, we are considering the integration of probabilistic aspects into the development of distributed algorithms and the refinement process. It appears that there are other proposals for dealing with probabilisitic choices in algorithmic systems and we cite the works of McIver and Morgan ([12]), and the approach of Hoang et al ([8, 3]) which is integrated into the Event B methodology. We choose graph coloring algorithms to illustrate our topic, because of their strong ties to probabilities. The probabilistic aspects of these algorithms have already been studied by several researchers, such as Métivier et al ([14]), who analysed a coloring algorithm using random bits exchanges between the vertices of a graph, Kubale et al ([9, 10]), who analysed coloring algorithms based on the use of random numbers for the choice of colors by the vertices, and Duffy et al ([6]), who proposed an algorithm in which probabilities are distributed over the set of colors a vertex can choose. In fact, there is no or little verification of the safety correctness of these algorithms. The main contribution is yet the analysis of complexity, which can be done later on our final models.

Summary of the paper - Section 2 contains our methodology and the development and study of the graph coloring problem. Section 3 concludes and discusses the issues of this problem and the development of probabilistic distributed algorithms.

2 The Graph Coloring Problem

2.1 Development methodology

The figure Fig.1 presents the methodology we used to develop our algorithms. We begin by developing an abstract model and end with three concrete models which can be translated into three different algorithms.

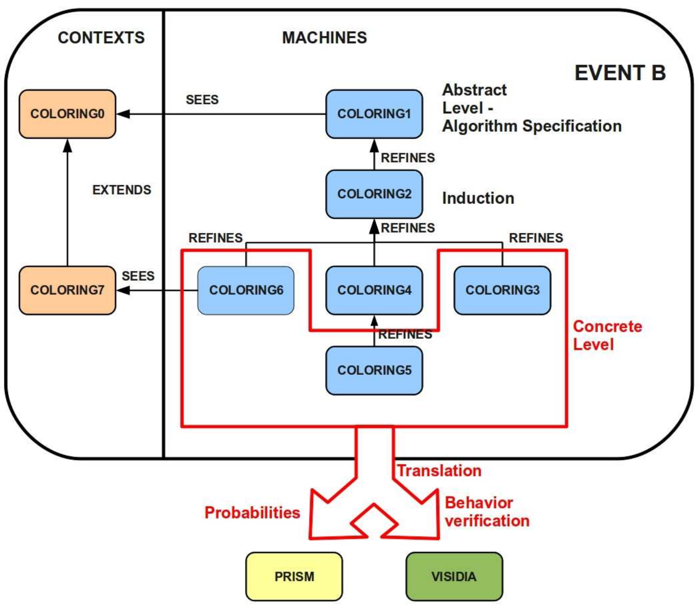

Fig. 1: The development methodology

2.2 Specification of the graph coloring problem

The specification of the graph coloring problem is done in the machine ***COLORING1***. A simple graph GRAPH is given over a set of vertices NODES. Using results of the graph theory, the vertices of GRAPH can be colored in such a way that no two adjacent vertices of GRAPH share the same color (see figure Fig.2). The vertices of GRAPH are colored by a statement in the machine.

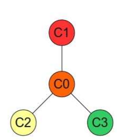

Fig. 2: A proper coloring of a graph

The next step is to define an inductive coloring function which associates different colors to the adjacent vertices of GRAPH.

2.3 Induction step : Computing the coloring function

In the machine ***COLORING2***, we define an inductive coloring function, which assigns the vertices their colors, which are different from the ones given to their neighbors (see figure Fig.3) .

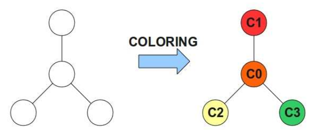

Fig. 3: The coloring function

This model does not take into account the notion of ill-chosen colors and multiple tentatives of colors choices. Next section will solve this question.

2.4 The vertex coloring algorithms

The algorithms defined in the machines ***COLORING3***, ***COLORING5***, ***COLORING6*** introduce the notion that a node might do multiple color choices before finding the right one (see figure Fig.4) .

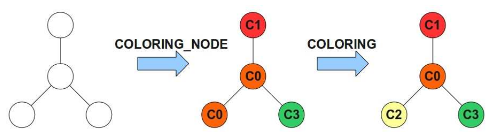

Fig. 4: The algorithms general behavior

COLORING3 - This model presents a synchronous algorithm divided into steps : in a first round, all the vertices which have no color or are not correctly colored choose a color, then in a second round, all the vertices which have chosen colors, update the graph by deleting the edges between them and their neighbours which have chosen different colors, and finally the vertices verify if they are still linked to some neighbours; if it is the case, they restart choosing colors otherwise they stop executing the algorithm.

COLORING5 and COLORING6 - These models present asynchronous algorithms. The behaviors of these algorithms are the same as the one defined in ***COLORING3***, except the fact that the vertices don't have to wait for all the others to choose their colors, update the links between them and their neighbours or restart choices.

Behavior verification - The visualisation and simulation of the algorithms are done with the software ViSiDiA ([5, 7, 15]).

2.5 Integration of Probabilistic Arguments

Our analysis shows that the probabilistic aspects of these algorithms lie within the random choice of a color by a vertice. Probabilities can be distributed over of the elements of the set of available colors for a vertice or over the set of the events which are parts of the global event *"choice of a color by a node"* : the event, when a node chooses a right color, happens with a certain probability, and the event when, it chooses a wrong one, has a certain probability attached to it, too. The next step is to analyse these aspects using probabilistic formal tools like PRISM ([11]).

3 Conclusion and Further Developments

The current stepwise development is focusing on algorithms wich require probabilistic arguments to achieve termination. We have derived these vertex coloring algorithms from an abstract specification of the graph coloring problem. They have in common the integration of possible errors during the choice of colors by vertices and this is the point where probabilistic arguments can be handled. As we have mentioned in the abstract, this paper is presenting preliminary elements of a global methodology for handling the correct-by-construction refinement-based approach applied to distributed algorithms and further work is needed to adapt technique and tools. It is also a first step towards our goal to obtain a development framework, integrating probabilistic arguments and refinement, for distributed algorithms. This framework can be an extension of the Event B method ([8, 3]) or, in our case, a combination of the Event B method and other formal methods (see figure Fig.5).

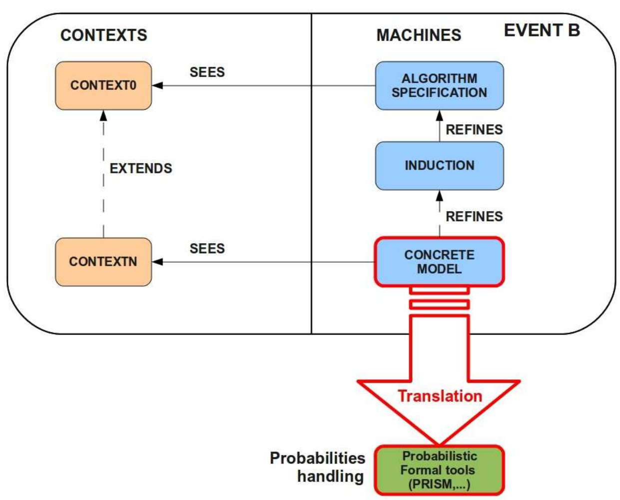

Fig. 5: A framework for the development of distributed algorithms

4 Acknowledgment

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