Class of Binary Generalized Goppa Codes Perfect in Weighted Hamming Metric

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Abstract. The class of the binary generalized Goppa codes is offered. It is shown that the codes of this class are on the Hamming bound constructed for a weighted metric.

1 Introduction

It is well-known that the problem of construction of good code for a given channel consists of two parts: a choice of a metric agreed with channel errors and search of a set of vectors with given metric properties as a correcting code. A great number of results of algebraic coding theory are codes for channels whose errors are agreed with the Hamming metric (HM). However, there are channels for which the error probability depends on a codeword position [1–4]. It means that the distribution of errors along codeword length is nonuniform. These channels are described by the model with nonuniform distribution of errors. Moreover, there are channels in which it is necessary to correct all errors from a certain set containing vectors with various configuration and weight [5,6]. The weighted Hamming metric (WHM) is a metric agreed with errors of such channels [7].

The paper presents a class of binary generalized Goppa codes constructed for the weighted Hamming metric (WHM). It is shown that parameters of the proposed class of codes are on the Hamming bound constructed for the WHM.

2 Weighted Hamming Metric

Definition 1. *[7] The distance between vectors* $\boldsymbol{a} = (a_1 a_2 \dots a_n)$ *and* $\boldsymbol{b} = (b_1 b_2 \dots b_n)$ *in the WHM is defined by a function* $d_{wH}(\boldsymbol{a}, \boldsymbol{b})$ *that is given as*

$$d_{wH}(\boldsymbol{a}, \boldsymbol{b}) = \sum_{i=1}^{n} w_i \cdot d(a_i, b_i) \ ,$$

where $w_i > 0,\ d(a_i, b_i) = 1$ *, if* $a_i \neq b_i$ *and* $d(a_i, b_i) = 0$ *, if* $a_i = b_i$.

The value w_i and vector $\boldsymbol{w} = (w_1, w_2, \dots, w_n)$ are called as a position i weight and a vector of weights of positions, respectively.

Definition 2. *A set* $\boldsymbol{v} = \{v_1, v_2, \ldots, v_l\}$, $l \leq n$ *consisting of all the different values of vector* $\boldsymbol{w}$ *components is called as a set of weights of the weighted Hamming metric.*

The length of the vector $\boldsymbol{a}$ can be written as $n = n_1 + n_2 + \ldots + n_l$, where n_i is a number of vector $\boldsymbol{a}$ positions with a weight v_i.

Let us consider a sphere of radius τ with a center in a vector $\boldsymbol{a} = (a_1 a_2 \ldots a_n)$ in a n-dimensional binary vector space V with the WHM given in it. It is clear that this sphere contains all the vectors $\boldsymbol{b} = (b_1 b_2 \ldots b_n)$, such that

$$d_{wH}(\boldsymbol{a}, \boldsymbol{b}) \leq \tau.$$

Then the volume of the sphere W_n^τ is determined by

$$W_n^\tau = \sum_{\{\tau_1,\ldots,\tau_l\}: \sum\limits_{i=1}^{l} \tau_i v_i \leq \tau} \prod_{i=1}^{l} \binom{n_i}{\tau_i} .$$

The Hamming bound in the WHM can be written in the following way:

$$M \cdot W_n^\tau \leq 2^n,$$

where $n = n_1 + n_2 + \ldots + n_l$ is a length of codeword, M is a number of codewords, d_{wH} is a minimum distance of the code in the WHM and $\tau = \lfloor \frac{d_{wH}-1}{2} \rfloor$.

Definition 3. *A binary block code with the length*

$$n = n_1 + n_2 + \ldots + n_l$$

and a set of weights $\boldsymbol{v} = (v_1, v_2, \ldots, v_l)$ *is called perfect in the weighted Hamming metric if its parameters are on the Hamming bound:*

$$M \cdot W_n^\tau = 2^n .$$

3 Special class of binary generalized $\boldsymbol{\Gamma(L, G)}$ codes

Generalized $\Gamma(L, G)$ codes [8] are natural generalization of the classical $\Gamma(L, G)$ codes introduced by V. Goppa [9]. They are determined by a Goppa polynomial $G(x)$ and set of numerators of codeword positions L.

In classical $\Gamma(L, G)$ codes, rational fraction functions $f_i(x) = \frac{1}{x - \alpha_i}$, $G(\alpha_i) \neq 0$ are elements of the set L.

In generalized $\Gamma(L, G)$ codes, rational fraction functions $f_i(x) = \frac{\nu_i(x)}{u_i(x)}$, where $u_i(x)$ is a polynomial over $GF(2^m)$, $\deg u_i(x) > 1$ and $(u_i(x), G(x)) = 1$, $(u_i(x), u_j(x)) = 1$, $\deg \nu_i(x) < \deg u_i(x) < \deg G(x)$, $(u_i(x), \nu_i(x))) = 1$ for all $i, j = 1, \ldots, n$, $i \neq j$ are elements of the set L. It should be noted that for binary $\Gamma(L, G)$ codes numerators of functions $f_i(x)$ are formal derivatives from denominators for both classical and generalized codes.

In this paper, we consider binary generalized $\Gamma(L,G)$ codes with an irreducible Goppa polynomial $G(x)$ and set $L = \left\{\frac{u_i'(x)}{u_i(x)}\right\}$, where $u_i(x)$ is a normalized irreducible polynomial over the field $GF(2^m)$, $\deg u_i(x) \le l$ and $u_i'(x)$ is a formal derivative of polynomial $u_i(x)$. Let us use the following Lemma in order to set the special class of codes that is considered here.

Lemma 1. *For any separable polynomial $\varphi(x) = \prod_i u_i(x)$ with coefficients from $GF(2^m)$ the following relation holds:*

$$\frac{\varphi'(x)}{\varphi(x)} = \sum_i \frac{u_i'(x)}{u_i(x)} ,$$

where $\varphi'(x), u_i'(x)$ are formal derivatives of $\varphi(x)$ and $u_i(x)$ respectively.

Now let us give a definition of a generalized binary $\Gamma(L,G)$ - code agreed with the WHM .

Definition 4. *[11, 12] The set of all binary vectors*

$$\boldsymbol{a} = (a_1^{(1)}\ a_2^{(1)} \dots a_{n_1}^{(1)}\ a_1^{(2)} \dots a_1^{(l)} \dots a_{n_l}^{(l)})$$

of length $n = n_1 + n_2 + \dots + n_l$ satisfying relation (1) is called a generalized $\Gamma(L,G)$ code agreed with the WHM with set of weights $v = \{r_1, r_2, \dots, r_l\}$.

$$\sum_{j=1}^{l}\sum_{i=1}^{n_j} a_i^{(j)} \frac{u_i^{(j)'}(x)}{u_i^{(j)}(x)} \equiv 0 \ mod\ G(x) , \tag{1}$$

where $\deg u_i^{(j)}(x) = r_j \le \deg G(x)$, $u_i^{(j)}(x)$ are irreducible polynomials of the degree r_j with coefficients from $GF(2^m)$, $(u_i^{(j)}(x), G(x)) = 1$ for all $j = 1, \dots, l;\ i = 1, \dots, n_j$, and $G(x)$ is a separable polynomial with coefficients from $GF(2^m)$.

In order to estimate the maximum power of the set L of the binary generalized $\Gamma(L,G)$ codes agreed with the WHM, let us give several statements necessary for providing the main result of this paper.

Theorem 1. *[10] The number of normalized polynomials of the degree l irreducible over the field $GF(2^m)$ is determined by the value*

$$I_2(l) = \frac{1}{l}\sum_{d|l} \mu(d) 2^{m\frac{l}{d}} ,$$

where $\mu(d)$ is the Möbius function:

$$\mu(d) = \begin{cases} 1 \ \ if\ d = 1 \ ; \\ (-1)^r if\ d\ is\ a\ product\ of\ r\ different\ prime\ numbers \ ; \\ 0 \ \ in\ all\ other\ cases. \end{cases}$$

It is well-known that the number of unitary separable polynomials of the first degree with coefficients from $GF(2^m)$ is $M^1_{2^m} = 2^m$.

Lemma 2. *[13]*
The number of unitary separable polynomials of the degree $l > 1$ with coefficients from the field $GF(2^m)$ is $M^l_{2^m} = 2^{ml} - 2^{m(l-1)}$.

Corollary 1. *The number of unitary separable polynomials with coefficients from the field $GF(2^m)$ whose degrees do not exceed l $(l > 1)$ is equal to*

$$N^l_{2^m} = \sum_{i=2}^{l} (2^{mi} - 2^{m(i-1)}) + 2^m = 2^{ml}.$$

The following statement can be obtained from Lemma 1 and Corollary 1 .

Proposition 1. *The number of different rational functions $\frac{f'(x)}{f(x)}$, where $1 \le \deg f(x) \le l$, $f'(x)$ is a formal derivative of $f(x)$ and $f(x)$ is an unitary separable polynomial with coefficients from $GF(2^m)$, is*

$$V^l_{2^m} = 2^{ml}.$$

4 Perfect linear binary codes in the weighted Hamming metric

Let us use definition (4) in order to present a special class of binary generalized $\Gamma(L,G)$ codes. Now, let $l = \tau$, where τ is a degree of the Goppa polynomial $G(x)$ of the binary generalized $\Gamma(L,G)$ code. In this case all polynomials of the degree up to τ that are irreducible over $GF(2^m)$ are chosen as denominators for different elements of the set L. The length of the generalized binary $\Gamma(L,G)$ code will be equal to n :

$$n = n_1 + n_2 + \ldots + n_\tau \ ,$$

where $n_1 = 2^m, \ n_2 = I_{2^m}(2) \ , \ldots , \ n_\tau = I_{2^m}(\tau) - 1$.

It means that the first n_1 codeword positions are numerated by all irreducible polynomials of the first degree $u_i^{(1)}(x)$. The next n_2 positions are numerated by all irreducible polynomials of the second degree $u_i^{(2)}(x)$ and so on. The last n_τ positions are numerated by all irreducible polynomials $u_i^{(\tau)}(x)$ of the degree τ with the exception of one that was chosen as Goppa polynomial $G(x)$.

Obviously, when presenting generalized $\Gamma(L,G)$ codes in the weighted Hamming metric the weight of the i-th code position w_i is determined by the degree r_i of the polynomial $u_i(x)$ used for numbering of this position:

$$w_i = r_i \ . \tag{2}$$

Let us determine the number of nonzero binary vectors contained in a sphere of radius τ provided that we consider the binary generalized $\Gamma(L,G)$ code whose position numerators are rational functions $\frac{u_i^{'(j)}(x)}{u_i^{(j)}(x)}$, where $u_i^{(j)}(x)$are all the different irreducible polynomials of the degree $r_{(j)} \le \tau$ with coefficients from $GF(2^m)$ and $G(x)$ is a irreducible polynomial of degree τ that has not been used for code position numbering. Let $\boldsymbol{e} = (e_1^{(1)} e_2^{(1)} \ldots e_{n_1}^{(1)} e_1^{(2)} \ldots e_1^{(\tau)} \ldots e_{n_\tau}^{(\tau)})$ be an error vector and $E(x)$ be a syndrome for this vector.

$$\sum_{j=1}^{\tau} \sum_{i=1}^{n_j} e_i^{(j)} \frac{u_i^{'(j)}(x)}{u_i^{(j)}(x)} \equiv E(x) \; mod \; G(x) \; , \tag{3}$$

Lemma 3. *Syndromes for various error vectors*

$$\boldsymbol{e} = (e_1^{(1)} e_2^{(1)} \ldots e_{n_1}^{(1)} e_1^{(2)} \ldots e_1^{(\tau)} \ldots e_{n_\tau}^{(\tau)}), \; wt_{wH}(\boldsymbol{e}) \le \tau$$

are different.

Proof. We will prove the lemma by contradiction. Assume that we found two different binary vectors

$$\begin{array}{l} \boldsymbol{e} = (e_1^{(1)} e_2^{(1)} \ldots e_{n_1}^{(1)} e_1^{(2)} \ldots e_1^{(\tau)} \ldots e_{n_\tau}^{(\tau)}), \, wt_{wH}(\boldsymbol{e}) \le \tau \\ \widehat{\boldsymbol{e}} = (\widehat{e}_1^{(1)} \widehat{e}_2^{(1)} \ldots \widehat{e}_{n_1}^{(1)} \widehat{e}_1^{(2)} \ldots \widehat{e}_1^{(\tau)} \ldots \widehat{e}_{n_\tau}^{(\tau)}), \, wt_{wH}(\widehat{\boldsymbol{e}}) \le \tau \end{array},$$

for which the equality

$$\sum_{j=1}^{\tau} \sum_{i=1}^{n_j} e_i^{(j)} \frac{u_i^{'(j)}(x)}{u_i^{(j)}(x)} \equiv \sum_{j=1}^{\tau} \sum_{i=1}^{n_j} \widehat{e}_i^{(j)} \frac{u_i^{'(j)}(x)}{u_i^{(j)}(x)} \equiv E(x) \; mod \; G(x) \; .$$

is true. This equality can be rewritten in the following form:

$$\sum_{j=1}^{\tau} \sum_{i=1}^{n_j} (e_i^{(j)} \oplus \widehat{e}_i^{(j)}) \frac{u_i^{'(j)}(x)}{u_i^{(j)}(x)} \equiv 0 \; mod \; G(x)$$

or

$$\frac{\varphi'_{e+\widehat{e}}(x)}{\varphi_{e+\widehat{e}}(x)} \equiv 0 \; mod \; G(x) \; , \tag{4}$$

$\deg \varphi'_{e+\widehat{e}}(x) < \deg \varphi_{e+\widehat{e}}(x) = wt_{wH}(\boldsymbol{e} \oplus \widehat{\boldsymbol{e}}) \le 2\tau$. Polynomial $\varphi'_{e+\widehat{e}}(x)$ is the formal derivative in $GF(2^m)$ and therefore it has nonzero coefficients only at even powers of x. It means that it can be represented as a square of some polynomial $\psi_{e+\widehat{e}}(x)$:

$$\varphi'_{e+\widehat{e}}(x) = (\psi_{e+\widehat{e}}(x))^2 \; ; \deg \psi_{e+\widehat{e}}(x) < \tau \; .$$

Hence we can rewrite equation (4) in the following form:

$$\frac{(\psi_{e+\widehat{e}}(x))^2}{\varphi_{e+\widehat{e}}(x)} \equiv 0 \; mod \; G(x) \; .$$

To make this equality true, it is necessary to obtain $\psi_{e+\widehat{e}}(x) \equiv 0 \; mod \; G(x)$, but it is impossible because $\psi_{e+\widehat{e}}(x) \neq 0$ and $\deg \psi_{e+\widehat{e}}(x) < \tau = \deg G(x)$.

Since the total number of different nonzero polynomials $E(x)$, $\deg E(x) < \tau$ with coefficients from $GF(2^m)$ is equal to $2^{m\tau} - 1$, and it is equal to the number of different binary vectors $\boldsymbol{e}$, $0 < wt_{wH}(\boldsymbol{e}) \leq \tau$ defined by the lemma condition then it follows that for any such vector $\boldsymbol{e}$ there exists an unique polynomial $E(x)$ satisfying equation (3).

Corollary 2. *In the weighted Hamming metric, in the sphere of radius τ the number of vectors*

$\boldsymbol{e} = (e_1^{(1)} e_2^{(1)} \ldots e_{n_1}^{(1)} e_1^{(2)} \ldots e_1^{(\tau)} \ldots e_{n_\tau}^{(\tau)})$, *with the length* $n = n_1 + n_2 + \ldots + n_\tau$,

where $n_1 = 2^m,\ n_2 = I_{2^m}(2)\ , \ldots,\ n_\tau = I_{2^m}(\tau) - 1$ *and* $v_1 = 1, v_2 = 2, \ldots, v_\tau = \tau$ *is equal to*

$$W_n^\tau = 2^{m\tau} \ .$$

The following statement can be stated by using Corollary 2 and Lemma 1,.

Theorem 2. *Binary generalized $\Gamma(L, G)$ codes with the Goppa polynomial $G(x)$ ($G(x)$ is an irreducible polynomial with coefficients from $GF(2^m)$ whose degree is equal to τ) and the set L*

$$L = \left\{ \frac{u_i^{(j)\prime}(x)}{u_i^{(j)}(x)} \right\}_{i=1,\ldots,n_j;\, j=1,\ldots,\tau}$$

where $u_i^{(j)}(x)$ is an irreducible polynomial of degree j with coefficients from $GF(2^m)$ and $G(x) \neq u_i^{(\tau)}(x)$ for all i,

are the perfect codes in the WHM, i.e., their parameters are on the Hamming bound:

$$2^k \cdot W_n^\tau = 2^n,$$

where $n = \sum\limits_{j=1}^{\tau} n_j$ is the codeword length, k is the dimension of the code and W_n^τ is the number of vectors with length n in a sphere of radius τ in the WHM.

We can obtain the following Corollary from this theorem and Lemma 3.

Corollary 3. *Binary generalized $\Gamma(L, G)$ code perfect in the WHM has the minimum distance $d_{\omega H} = 2\tau + 1$. It can correct any $t \leq \sum\limits_{j=1}^{\tau} t_j$ errors, where $\sum\limits_{j=1}^{\tau} j t_j \leq \tau$ and t_j is the number of errors on the n_j positions of the codeword.*

Remark 1. Note that the binary generalized $\Gamma(L, G)$ codes perfect in the WHM can be decoded as generalized $\Gamma(L, G)$ codes. They are decoded by any of the algorithms used for alternant codes, for instance, such as the Euclidian algorithm [10].

5 Code example

Let us consider a logical net similar to that given in [6] whose design is shown in Figure 1. There are n inputs $x_1, x_2, \ldots, x_n$ and 9 outputs $y_1, y_2, \ldots, y_9$. Let us take that only one unit of the net can function erroneously at every time moment [6]. The faulty operation of unit B_1 leads to an error in output signals y_1 and y_2 so that the corresponding error vector is equal to $e_1 = (110000000)$. An error in unit B_2 leads to errors in output signals y_1 and y_3 and the error vector $e_2 = (101000000)$. Errors in other units will lead to similar results. For instance, an error in unit B_7 gives an error in signal y_1 and the error vector will be $e_7 = (100000000)$. Thus, the set of possible error vectors is

$$\begin{aligned} F &= \{e_1, e_2, \ldots, e_{15}\} \\ &= \{(110000000), (101000000), (100100000), (011000000), (010100000), \\ &\quad (001100000), (100000000), (010000000), (001000000), (000100000), \\ &\quad (000010000), (000001000), (000000100), (000000010), (000000001)\} \, . \end{aligned} \tag{5}$$

Let us construct a code correcting the total set of error vectors (5) for the logical net. We choose the generalized binary (L, G) code with $n_1 = 4,\ n_2 = 5,\ k = 5,\ d_{wH} = 5$ that is perfect in the WHM. (It is clear that this code is on the upper bound for linear codes [14]). Any polynomial of the second degree irreducible over $GF(2^2)$ with coefficients from the same field can be chosen as a Goppa polynomial for this code. The total number of such polynomials of the second degree is equal to $\frac{(2^2)^2 - 2^2}{2} = 6$. One of these polynomials is used as the Goppa polynomial $G(x)$, whereas the rest polynomials form denominators of elements of the set L. Besides, four rational functions with denominators of the first degree $\left\{\frac{1}{x-1}, \frac{1}{x-\alpha}, \frac{1}{x-\alpha^2}, \frac{1}{x}\right\}$ are used in the set L, where α is a primary element of $GF(2^2)$. Let us find the number of vectors in the sphere of radius 2

$n_1 = 4$	$n_2 = 5$	$n = 9$
$v_1 = 1$	$v_2 = 2$	
t_1	t_2	$t = t_1 + t_2$
0	≤ 1	≤ 1
≤ 2	0	≤ 2

Table 1. Binary generalized $\Gamma(L, G)$ code with $(n_1 = 4,\ n_2 = 5,\ k = 5,\ d_{wH} = 5)$

in the weighted Hamming metric for this code:

$$1 + \binom{1}{4} + \binom{2}{4} + \binom{1}{5} = 1 + 4 + 6 + 5 = 16 = 2^4 \, .$$

The number of codewords of this code is equal to 2^5. Since the spheres of radius 2 whose centers are in any two codewords of such code do not intersect, then

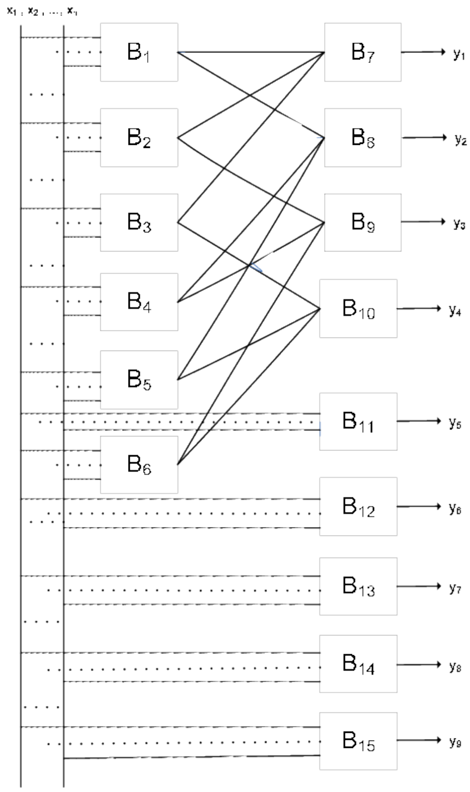

Fig. 1. Logical net

the total number of vectors in such nonintersecting spheres is

$$2^5 \cdot 2^4 = 2^9 \ ,$$

which corresponds to the total number of various binary vectors of length 9.

Table 5 shows all known binary generalized $\Gamma(L, G)$ codes perfect in the WHM with the length less than 1500.

6 Notes and comments

In conclusion, it should be noted that the above considered codes are similar to the binary Hamming codes for the HM. The open problem is the existence of perfect codes in the WHM that are similar to non-binary Hamming codes and the Golay codes.

n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}	n_{12}	n_{13}	n	k	$d_{\omega H}$	d	d_{Ub}
2	1	1	0	0	0	0	0	0	0	0	0	0	4	1	7	4	4
2	1	2	2	0	0	0	0	0	0	0	0	0	7	3	9	3	4
2	1	2	3	5	0	0	0	0	0	0	0	0	13	8	11	3	4
2	1	2	3	6	8	0	0	0	0	0	0	0	22	16	13	3	4
2	1	2	3	6	9	17	0	0	0	0	0	0	40	33	15	3	4
2	1	2	3	6	9	18	29	0	0	0	0	0	70	62	17	3	4
2	1	2	3	6	9	18	30	55	0	0	0	0	126	117	19	3	4
2	1	2	3	6	9	18	30	56	98	0	0	0	225	215	21	3	4
2	1	2	3	6	9	18	30	56	99	185	0	0	411	400	23	3	-
2	1	2	3	6	9	18	30	56	99	186	334	0	746	734	25	3	-
2	1	2	3	6	9	18	30	56	99	186	335	629	1376	1363	27	3	-
4	5	0	0	0	0	0	0	0	0	0	0	0	9	5	5	3	3
4	6	19	0	0	0	0	0	0	0	0	0	0	29	23	7	3	4
4	6	20	59	0	0	0	0	0	0	0	0	0	89	81	9	3	4
4	6	20	60	203	0	0	0	0	0	0	0	0	293	283	11	3	-
4	6	20	60	204	669	0	0	0	0	0	0	0	963	951	13	3	-
8	27	0	0	0	0	0	0	0	0	0	0	0	35	29	5	3	3
8	28	167	0	0	0	0	0	0	0	0	0	0	203	194	7	3	4
8	28	168	1007	0	0	0	0	0	0	0	0	0	1211	1199	9	3	-
16	119	0	0	0	0	0	0	0	0	0	0	0	135	127	5	3	3
16	120	1359	0	0	0	0	0	0	0	0	0	0	1495	1483	7	3	-
32	495	0	0	0	0	0	0	0	0	0	0	0	527	517	5	3	-

Table 2. All known binary generalized $\Gamma(L, G)$ codes with the length less than 1500 perfect in the WHM. Positions weights are equal to $v_i = i$ on length n_i, d is the minimum distance in the HM, d_{Ub} is the upper bound for a minimum distance of the linear binary (n, k, d) code in HM [14].

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