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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

***A Theoretical Model for Analyzing the Impact of  
Jitter-based Techniques on Flooding over Wireless  
Ad Hoc Networks***

Juan Antonio Cordero  
Philippe Jacquet  
Emmanuel Baccelli

**N° 7701 — version 2**

initial version Juillet 2011 — revised version Février 2012

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***R**apport  
de recherche*



# A Theoretical Model for Analyzing the Impact of Jitter-based Techniques on Flooding over Wireless Ad Hoc Networks

Juan Antonio Cordero  
Philippe Jacquet  
Emmanuel Baccelli

Thème : Réseaux et télécommunications  
Équipe-Projet Hipercom

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**Abstract:** Jitter is used in wireless ad hoc networks to reduce the number of packet collisions and the number of transmissions. This is done by scheduling random back-off for each packet to be transmitted and by piggybacking multiple packets in a single transmission. This technique has been standardized by the IETF in RFC 5148. This paper investigates on the impact of the standardized jitter mechanism on network-wide packet dissemination – i.e. flooding, an important component for many protocols used today. A novel analytical model is introduced, capturing standard jitter traits. From this model is derived accurate characterization of the effects of jittering on flooding performance, including the additional delay for flooded packets on each traversed network interface, the reduction of the number of transmissions over each network interface, and the increased length of transmissions, depending on jitter parameters. This paper also presents an analysis of the use of jitter in practice, over an 802.11 wireless link layer based on CSMA. The analytical results are then validated via statistical discrete event simulations. The paper thus provides a comprehensive overview of the impact of jittering in wireless ad hoc networks.

**Key-words:** mobile; ad hoc; routing; jitter; flooding; network; model; probability; theory; analysis; standard; wireless

# A Theoretical Model for Analyzing the Impact of Jitter-based Techniques on Flooding over Wireless Ad Hoc Networks

**Résumé :** Le délai aléatoire, ou jitter, est utilisé dans les réseaux ad hoc pour réduire le numéro de collisions de paquet, aussi bien que le numéro de transmissions. Cela se fait en ajoutant un back-off aléatoire avant la retransmission de chaque paquet, et en envoyant plusieurs paquets d'une seule fois, en une seule transmission (piggybacking). Cette technique est standardisée par l'IETF au RFC 5148. Ce rapport analyse l'usage et l'impact du mécanisme du jitter dans la dissémination de paquets à travers le réseau (flooding), qui est un composant importante de plusieurs protocoles, en particulier protocoles de routage, couramment utilisées aujourd'hui. Le rapport propose un nouvel modèle analytique qui capture et décrit les traits et caractéristiques principales du jitter. Depuis ce modèle, le rapport déduit l'effet du jittering sur la performance de l'opération de dissémination périodique ou flooding, notamment le délai additionnel souffert par les paquets disséminés à travers le réseau, la réduction du nombre de retransmissions par interface et la longueur des paquets retransmis, en fonction des paramètres du mécanisme de jitter. Le rapport étudie aussi l'usage pratique du jitter dans des réseaux sans fils, au-dessus de technologies 802.11, basées sur CSMA. Les résultats analytiques obtenus à travers de ce modèle théorique sont validés par moyen de simulations à événements discrets. En somme, le rapport offre une perspective générale sur l'effet des techniques de jitter dans des réseaux ad hoc sans fils.

**Mots-clés :** mobile; ad hoc; routage; jitter; gigue; flooding; dissémination; réseau; modèle; probabilité; théorie; analyse; standard; sans fils

## 1 Introduction

Periodic and quick network-wide packet dissemination, *i.e.*, flooding, is fundamental to many protocols used in today's Internet. Several flooding techniques exist [16] [24] [14] the simplest one relying on the principle that each node in the network forwards a flooded packet once – the first time it receives this packet. In wireless multi-hop ad hoc networks, flooding is an essential component of some of the most prominent routing protocols, such as OLSR [15], MANET extensions of OSPF [2] [5] [7] and AODV [13].

Due to the characteristics of the shared wireless medium [4], nodes in ad hoc networks must often forward flooded packets on the same interface they were received on. Upon reception of a flooded packet, nearby nodes are thus likely to simultaneously forward the packet on the shared wireless medium, and thus systematically cause packet collisions.

In order to reduce the number of such collisions in a distributed fashion, random back-off times are independently scheduled by each node before each transmission, which aims at avoiding synchronized wireless medium access. Such a mechanism, called *jitter* or *jittering*, was standardized by the IETF in RFC 5148 [10]. Jitter thus decreases the number of collisions at the price of increased delay.

During the time a node waits before transmitting, additional flooded packets may be received. According to RFC 5148, these packets are then buffered and piggybacked in the node's next transmission. This jittering technique also decreases the number of transmissions, at the price of longer transmissions, *i.e.* bigger packets.

Unintended jitter has been widely studied, both theoretically and based on experimental analysis, in the context of real-time services (such as for voice transport or video streaming) for several networking scenarios, in particular for ATM [20] [22] or IP wired and wireless networks [17] [12] [11].

Deliberate jitter was initially used in ALOHA and CSMA, which have been widely studied in the literature [23] [8]. The optimal jitter range has been studied experimentally in [18], while [3] proposed an analytical model for broadcast transmissions, taking a network-wide approach to describe the relationship between jitter range and probability of transmission without collision, and evaluated the use of jitter at different layers.

This memorandum studies the use of jitter techniques specified by the IETF in RFC 5148 [10], in the context of flooding over multi hop wireless networks. An analytical model is introduced, and several results are derived concerning incurred delays, transmission rates reduction and packet size increase. These results offer a comprehensive view of the impact of jitter on flooding performance.

The remainder of this memorandum is organized as follows. Section 2 describes the jittering techniques for preventing packet collisions in flooding. Section 3 presents an analytical model of the flooding operation in a wireless router. Section 4 presents the main aspects of the analysis of the impact of random delay in packet forwarding. Section 5 discusses the obtained results and puts them in the context of the use of jitter over 802.11-based links. Section 6 validates analytical results through simulations. Finally, section 7 concludes the memorandum .

## 2 The Jitter Mechanism for Flooding

This section details the use of jittering techniques, as specified in RFC 5148 [10], in the context of classical flooding, where each node in the network forwards a flooded packet once, the first time the packet is received. In this context, packet collisions occur when two neighboring nodes forward the same packet, immediately after its reception, as illustrated in Figure 1. It is worth to note that collisions in flooding addressed in this memorandum are systematic, *i.e.*, they come deterministically from the fact that two or more nearby routers take the same decision (to forward a flooded packet) in reaction to the same event (the reception of that packet). Prevention of these collisions, or at least reduction of them to random events with low probability, becomes thus a central issue to be handled for flooding in ad hoc networks.

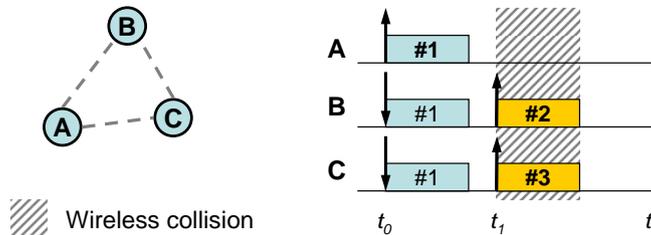


Figure 1: Wireless collision caused by concurrent flooding retransmissions. *The forwarding of a flooded packet (#1) by router A, received by B and C, causes simultaneous transmissions by routers B and C (#2 and #3), to forward the flooded packet, which cause a wireless collision.*

RFC 5148 [10] specifies techniques for reducing packet collisions occurrences. When an interface receives a message to be forwarded, a jitter value (denoted  $t$  throughout this memorandum) is selected randomly with a uniform distribution in the interval  $[0, J_m]$ , where  $J_m$  is the maximum jitter value (named by *MAXJITTER* in RFC 5148). According to the specification, such jitter value may be used for three cases of message transmission: periodic messages, externally triggered messages, and message forwarding. This memorandum focuses on the impact of jitter in message forwarding. In the following, we will consider messages disseminated network-wide in flooded packets that may contain one or more messages. The motivation for using jittering techniques in this case is therefore two-fold: (a) to reduce the number of wireless collisions by spreading message transmissions over time, and (b) to reduce the number of transmissions by piggybacking several messages in a single packet.

A wireless interface that receives a packet may decide to forward some of the messages contained in this packet. In this case, the interface assigns a jitter value to the messages to be forwarded – the same value for all messages belonging to the same packet – and schedules their transmission after the expiration of the jitter time-out. A wireless interface may also itself generate messages to be flooded. Such self-generated messages are scheduled for immediate transmission, which is equivalent to assigning them a jitter equal to zero. Then, when a transmission is scheduled, all buffered messages waiting to be transmitted – that were either received from other interfaces in the mean time, or self-generated–

are sent in a single packet. The flowchart in Figure 2 summarizes this procedure, in which three elements can be pointed out:

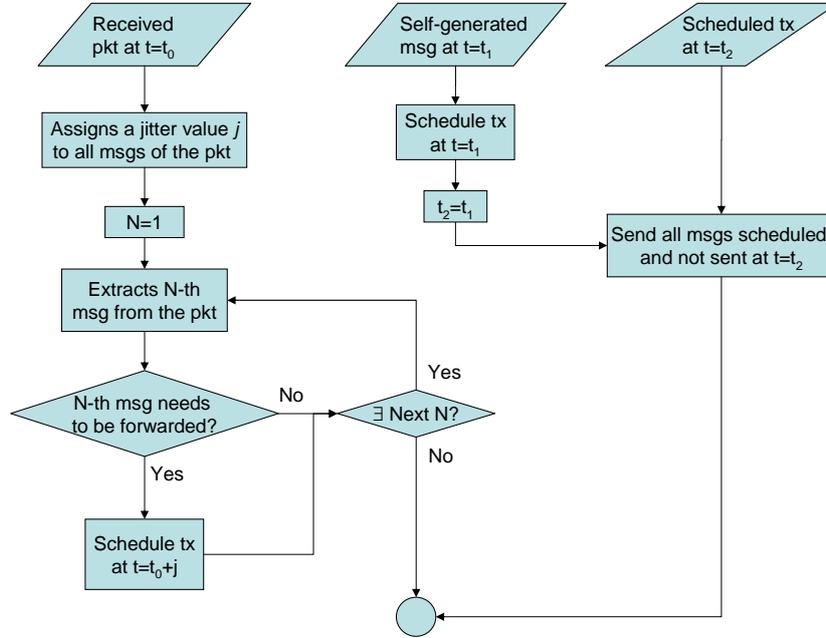


Figure 2: Forwarding algorithm with jitter.

- Effective and scheduled time of transmission.** Messages are forwarded with a delay shorter or equal to their assigned jitter value, given the fact that *all* pending transmissions are performed together when the jitter of *any* pending message expires. The gap between assigned delay and effective delay depends on the arrival rate of packets with messages to be forwarded.
- Immediate flooding of self-generated messages.** The fact that self-generated messages are sent immediately also contributes to the gap between assigned and effective delays. Self-generated message rate, packet reception rate and jitter value bounds (*MAXJITTER*) are therefore factors that impact the effective delay of forwarded messages. If the self-generated message rate increases significantly, it may become the dominating factor and render irrelevant changes in the interval for jitter values.
- Impact on packet rate.** Since forwarded packets may contain messages from one or more received packets, the use of jittering techniques may lead to a reduction in the rate of flooded packets – even in cases where an interface forwards all the message it receives. In particular, a wireless interface forwards packets at a lower rate than it receives packets to be forwarded. This is, however, at the expense of increasing the size of transmitted packets, as they contain a growing number of messages.

The analysis presented in this memorandum evaluates the impact of the above three elements by way of a probabilistic theoretical model.

	Packets	Messages
Received to fwd	$\lambda_{in}$	$\gamma_{in}$
Self-generated	$\lambda_g$	$\gamma_g$
Sent	$\lambda_{out}$	$\gamma_{out}$

Table 1: Traffic model variables.

### 3 System Model

This section describes the main parameters and assumptions under which the jitter mechanism is evaluated. Section 3.1 defines the scope of this model and the variables used to parametrize operation of the jitter mechanism. Section 3.2 presents the types of traffic considered in the model and describes the assumptions over them.

#### 3.1 Model Scope and Parameters

The model presented in this section examines the use of the jitter mechanism in a particular wireless router (denoted throughout this memorandum as a *node*) attached to a network, that participates in the flooding of traffic from other nodes and also generates traffic to be flooded over the network. It is assumed that all nodes in the network have the same configuration of the jitter mechanism. That implies that jitter values, denoted by  $t$  throughout this memorandum, are selected within the same interval  $[0, J_m]$  and with the same distribution, where  $J_m$  is the maximum value for the jitter (denominated *MAXJITTER* in RFC 5148 [10]).

#### 3.2 Traffic Model and Assumptions

Nodes participate in flooding by generating, receiving and forwarding *messages*. These messages are sent through the network in *packets*, each packet containing one or more messages.

Three types of traffic are distinguished:

- traffic received by the node to be forwarded (in-traffic),
- traffic generated by the node (self-traffic), and
- traffic sent by the node (out-traffic).

There may be an additional type of traffic: traffic received by the node, but not forwarded. For the purposes of this memorandum, this non-forwarded traffic is not relevant, and is thus not considered. For convenience, it will be therefore considered that all packets received are to be forwarded. Table 1 displays the variables used for describing the traffic rates in terms of messages per second ( $\gamma$ ) and packets per second ( $\lambda$ ), and Figure 3 illustrates the traffic model for a particular node.

Packet arrivals to the node (either self-generated or received from other nodes) are modeled as punctual homogeneous Poisson processes.

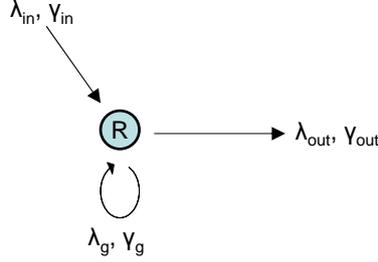


Figure 3: Node model.

## 4 Analysis

This section presents the theoretical results of the analysis based on the previously presented jitter model. Results are presented for a general distribution of the random variable for jitter values,  $T_j$ , and then particularized for the case of uniform distribution, specified in RFC 5148, *i.e.*:

$$\begin{cases} f_{T_j}(t) &= \frac{\mathbf{1}_{[0, J_m]}(t)}{J_m} \\ F_{T_j}(t) &= \frac{t}{J_m} \mathbf{1}_{[0, J_m]}(t) + \mathbf{1}_{(J_m, +\infty)}(t) \end{cases} \quad (1)$$

Section 4.1 indicates the relationship between the different types of considered traffic, both in terms of messages and packets. The analysis focuses on the *collecting phase* of a node, which can be defined as follows:

- The **collecting phase** of a node using jitter for flooding over an ad hoc network is the period between the first in-packet arrival after an out-packet transmission, and the following out-packet transmission. Duration of this length is bounded by the jitter value assigned to such first in-packet.

Section 4.2 analyzes the average length of the collecting phase, denoted by  $D(t)$ , where  $t$  is the jitter value assigned to in-packet triggering the phase. Figure 4 illustrates the notion of collecting phase in a node reception/transmission timeline.

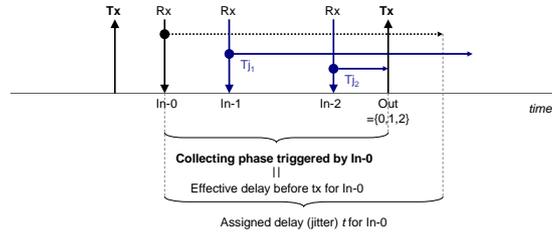


Figure 4: Collecting phase of a node participating in flooding.

The average number of packets received during the collecting phase, as well as the impact of jitter in out-packet size, are studied in section 4.3. Based on these results, section 4.4 describes the out-packet rate,  $\lambda_{out}$ , in function of the in-packet and the self-packet rates, and checks its consistency by examining its asymptotic behavior when one of the two input traffic components (in-traffic

and self-traffic) dominates the other. Finally, section 4.5 presents and describes the notion of cumulated delay of in-packets of a collecting phase, and computes the average delay for an in-packet, depending on the jitter interval.

#### 4.1 Message and Packet Rates

This section describes the relationship between message and packet rates received and sent by a node. Every message that a node sends to the network (out-message) has been either received to be forwarded (in-message), or created by the node to describe its own topology (self-message). Therefore, message rates satisfy the following relationship:

$$\gamma_{out} = \gamma_{in} + \gamma_g \quad (2)$$

Packets contain one or more messages. For consistency, it is assumed that a self-generated packet contains one and only one self-generated message, that is:

$$\lambda_g = \gamma_g \quad (3)$$

The relationship among packet rates ( $\lambda_{out}$ ,  $\lambda_{in}$ ,  $\lambda_g$ ) depends on the jitter mechanism. In-messages may be forwarded by way of (a) out-packets that contain only other in-messages, or (b) out-packets that contain one (and only one) self-generated message. The rate of out-packets in case (b) is then exactly  $\lambda_g$ . Out-packets in case (a) correspond to in-packets for which no self-traffic is generated while waiting for transmission. As out-packets in case (a) contain the messages from all the in-packets received, but not yet forwarded, the rate of out-packets in case (a) is significantly lower than the in-packet rate. If  $M^*$  denotes the random variable indicating the average number of in- and self-packets whose messages are included in a transmitted out-packet, the out-packet rate can be expressed as follows:

$$\lambda_{out} = \frac{\lambda_{in} + \lambda_g}{\mathbb{E}\{M^*\}} \quad (4)$$

Note that, as  $M^* \geq 1$ , the out-packet rate is always smaller or equal than the addition of in-packet and self-packet rates. Consequently, the average number of messages per out-packet (and thus the out-packet size) may increase with respect to the number of messages per in-packet. This variation in packet size can be computed as follows:

$$\frac{\gamma_{out}/\lambda_{out}}{\gamma_{in}/\lambda_{in}} = \frac{(\gamma_{in} + \lambda_g)/(\lambda_{in} + \lambda_g)}{\gamma_{in}/\lambda_{in}} \mathbb{E}\{M^*\} \quad (5)$$

If self-packet rate is lower than in-packet rate ( $\lambda_g \ll \lambda_{in}$ ), then:

$$\frac{\gamma_{out}/\lambda_{out}}{\gamma_{in}/\lambda_{in}} \approx \mathbb{E}\{M^*\} \quad (6)$$

Random variable  $M^*$  and its mean  $\mathbb{E}\{M^*\}$  are computed and examined in detail in section 4.3.

## 4.2 Average Duration of the Collecting Phase

This section studies the relationship between jitter values and length of collecting phases. Intuitively, the collecting phase is longer as the jitter value of the triggering in-packet increases, and gets shorter as the in-packet and self-packet rates ( $\lambda_{in}$  and  $\lambda_g$ , respectively) increase.

Let  $D$  be the random variable that describes the average duration of the collecting phase triggered by an in-packet arrival. *Theorem 1* defines the function  $D(t)$ , which corresponds to the value that  $D$  takes when the triggering in-packet is assigned a jitter  $t$ , in function of the maximum jitter value,  $J_m$ , and the traffic rates.

**Theorem 1.** *Let  $D(t)$  be the average duration of the accumulating phase, with  $t \in [0, J_m]$  being the scheduled time of retransmission of such first in-packet and  $J_m$  being the maximum jitter value. Then,  $D(t)$  satisfies the following ODE:*

$$D''(t) = (-\lambda_{in}f_{T_j}(t) - \lambda_g)D'(t) \quad (7)$$

*Proof.* Given a scheduled jitter value  $t$  for the first in-packet, the effect of events happening in  $dt$  in the average duration  $D$  is examined. For sufficiently small values of  $dt$ , only one Poisson event (an in-packet arrival, with rate  $\lambda_{in}$ ; or a self-generated packet, with rate  $\lambda_g$ ) may occur. An in-packet arrival at  $dt$  (with probability  $\lambda_{in}dt$ ) may modify the duration  $D(t)$  if the scheduled jitter  $T_j$  of the arrived packet is lower than the scheduled time of retransmission  $t$ ; a self-generated packet arrival within at  $dt$  (with probability  $\lambda_g dt$ ) implies that the duration  $D(t)$  becomes equivalent to the duration of the phase for a scheduled time  $dt$ . When no in- or self-packets arrive at  $dt$ , duration  $D(t)$  is equivalent to the duration obtained by waiting a  $dt$  interval and then scheduling retransmission after an interval  $t - dt$ . This is described formally in the following transition equation:

$$\begin{aligned} D(t) = & \lambda_{in}dt \left( P(T_j > t)D(t) + \int_0^t f_{T_j}(x)D(x)dx \right) + \\ & + \lambda_g dt D(dt) + (1 - (\lambda_{in} + \lambda_g)dt)(D(t - dt) + dt) \end{aligned}$$

Then,

$$\begin{aligned} D(t) - D(t - dt) = & \lambda_{in}dt (P(T_j > t)D(t) + \\ & + \int_0^t f_{T_j}(x)D(x)dx) + \\ & + \lambda_g dt D(dt) - \\ & - (\lambda_{in} + \lambda_g)dt(D(t - dt) + dt) \end{aligned}$$

And dividing over  $dt$ ,

$$\begin{aligned} \frac{D(t) - D(t - dt)}{dt} = & \lambda_{in} (P(T_j > t)D(t) + \\ & + \int_0^t f_{T_j}(x)D(x)dx) + \lambda_g D(dt) + \\ & + (\lambda_{in} + \lambda_g)(D(t - dt) + dt) \end{aligned}$$

Dividing over  $dt$  and for  $dt \rightarrow 0$ , taking into account that  $D(dt \rightarrow 0) \rightarrow 0$  by definition of  $D$ , the following differential equation arises:

$$\begin{aligned}
D'(t) &= \lambda_{in} \left( P(T_j > t)D(t) + \int_0^t f_{T_j}(x)D(x)dx \right) - \\
&\quad - (\lambda_{in} + \lambda_g)D(t) + 1 = \\
&= \lambda_{in} \left( (P(T_j > t) - 1)D(t) + \int_0^t f_{T_j}(x)D(x)dx \right) - \\
&\quad - \lambda_g D(t) + 1 = \\
&= \lambda_{in} \left( -F_{T_j}(t)D(t) + \int_0^t f_{T_j}(x)D(x)dx \right) - \\
&\quad - \lambda_g D(t) + 1
\end{aligned}$$

As  $F_X(t) = \int_0^t f_X(x)dx$ ,

$$D'(t) = \lambda_{in} \int_0^t f_{T_j}(x)(D(x) - D(t))dx - \lambda_g D(t) + 1$$

Differentiating this expression over  $t$ :

$$D''(t) = \frac{d}{dt} \left[ \lambda_{in} \int_0^t f_{T_j}(x)(D(x) - D(t))dx \right] - \lambda_g D'(t) \quad (8)$$

Where the derivative in brackets, denoted  $I_1$ , can be calculated as follows:

$$\begin{aligned}
I_1 &= \frac{d}{dt} \left[ \lambda_{in} \int_0^t f_{T_j}(x)(D(x) - D(t))dx \right] = \\
&= \frac{d}{dt} \left[ \lambda_{in} \left( \int_0^t f_{T_j}(x)D(x)dx - \right. \right. \\
&\quad \left. \left. - \int_0^t f_{T_j}(x)D(t)dx \right) \right] = \\
&= \lambda_{in} \frac{d}{dt} \left[ \int_0^t f_{T_j}(x)D(x)dx \right] - \\
&\quad - \lambda_{in} \frac{d}{dt} \left[ D(t) \int_0^t f_{T_j}(x)dx \right] = \\
&= \lambda_{in} f_{T_j}(t)D(t) - \lambda_{in} \frac{d}{dt} [D(t)F_{T_j}(t)] = \\
&= \lambda_{in} f_{T_j}(t)D(t) - \lambda_{in} D'(t)F_{T_j}(t) - \lambda_{in} D(t)f_{T_j}(t) = \\
&= -\lambda_{in} D'(t)F_{T_j}(t)
\end{aligned}$$

Then, replacing  $I_1$  in equation (8) leads to the ordinary differential equation (ODE) of order 2:

$$D''(t) = (-\lambda_{in} F_{T_j}(t) - \lambda_g)D'(t) \quad (9)$$

□

**Corollary.** *If jitter values are distributed uniformly within  $[0, J_m]$ , according to (1), the differential equation (7) has the following solution for  $D(t)$ :*

$$D(t) = \sqrt{\frac{2J_m}{\lambda_{in}}} e^{-\frac{\lambda_g}{2\lambda_{in}} J_m} \int_{\sqrt{\frac{J_m}{2\lambda_{in}}}\lambda_g}^{\sqrt{\frac{J_m}{2\lambda_{in}}}\left(\lambda_g + \frac{\lambda_{in}t}{J_m}\right)} e^{-s^2} ds \quad (10)$$

*Proof.* The result is immediate by imposing initial conditions  $D(0) = 0$ ,  $D'(0) = 1$  and assuming an uniform distribution for jitter values within  $[0, J_m]$ .  $\square$

### 4.3 Arrivals during the Collecting Phase and Packet Size

An out-packet sent at the completion of a collecting phase contains the messages included in the in-packet that triggered the phase, and the messages included in the in-packets arrived within the phase. In case that a self-packet is generated within the collecting phase, that terminates the phase and causes the transmission of the corresponding out-packet. *Theorem 2* proves that the average number of in- and self-packets included in an out-packet sent when a collecting phase terminates, denoted by  $M$ , follows the Poisson law.

**Theorem 2.** *Let  $M(t)$  be the average number of packets whose messages are transmitted together after the completion of a collecting phase started by an in-packet with initial jitter  $t \in [0, J_m]$ . Then, the expression of  $M(t)$  is as follows:*

$$M(t) = 1 + (\lambda_{in} + \lambda_g)D(t) \quad (11)$$

*Proof.* When an in-packet, with initial jitter  $t$ , arrives to an interface and starts a collecting phase, the number of packets whose messages are sent together in the next transmission of such interface is:

$$M(t) = 1 + N(t)$$

Where  $N(t)$  corresponds to the number of packets (in- and self-packets) arrived to the interface during the collecting phase – not including the in-packet that started such phase. For sufficiently small values of  $dt$ , the arrival of a self-generated message in  $dt$  implies that the out-packet is immediately sent, with only such self-message and the messages from the starting in-packet. In case that an in-packet arrives during  $dt$ , the number of packets included in the transmitted out-packet is incremented by one. The transition equation for  $N(t)$  is therefore as follows:

$$\begin{aligned} N(t+dt) &= \lambda_g dt + \lambda_{in} dt (P(T_j > t)(N(t) + 1) + \\ &\quad + \int_0^t f_{T_j}(x)(N(x) + 1)dx) + \\ &\quad + (1 - (\lambda_g + \lambda_{in})dt)N(t) \end{aligned}$$

Which leads to the following ODE:

$$N'(t) = \lambda_g + \lambda_{in} \left( 1 + (1 - F_{T_j}(t))N(t) + \int_0^t f_{T_j}(x)N(x)dx \right) - (\lambda_g + \lambda_{in})N(t)$$

Differentiating over  $t$ :

$$\begin{aligned} N''(t) &= \lambda_{in} \left( -f_{T_j}(t)N(t) + (1 - F_{T_j}(t))N'(t) + f_{T_j}(t)N(t) \right) - (\lambda_g + \lambda_{in})N'(t) = \\ &= \lambda_{in}(1 - F_{T_j}(t))N'(t) - (\lambda_g + \lambda_{in})N'(t) = \\ &= -(\lambda_g + \lambda_{in}F_{T_j}(t))N'(t) \end{aligned}$$

Which is the same ODE as (9). Assuming the following initial conditions:

$$\begin{cases} N(0) &= 0 \\ N'(0) &= \lambda_{in} + \lambda_g \end{cases}$$

The solution is therefore:

$$N(t) = (\lambda_{in} + \lambda_g)D(t)$$

□

$M$  and  $N$  count the average number of self- and in-packets contained in an out-packet sent at the completion of a collecting phase, including ( $M$ ) or excluding ( $N = M - 1$ ) the in-packet that triggers the collecting phase. Note that  $M$  and  $N$  are random variables because the value they take depend on the jitter value assigned to in-packet that trigger the collecting phase – which is itself computed randomly, according to the law of the random variable  $T_j$ . When this initial in-packet is assigned a jitter value  $t$ , random variables  $M$  and  $N$  take values  $M(t)$  and  $N(t)$ , respectively. Function  $N(t) = (\lambda_{in} + \lambda_g)D(t)$  is introduced in the proof of *Theorem 2*. Similarly, the average number of additional in-packets within the collecting phase of an in-packet with assigned jitter  $t$  can be computed as:

$$N_i(t) = \lambda_{in}D(t) \tag{12}$$

Random variable  $M^*$  counts the number of self- and in-packets contained in an out-packet. It is worth to observe that this definition is more general than the definition of  $M$  – it takes into account any out-packet, not only out-packets sent at the completion of a collecting phase triggered by an in-packet arrival. Out-packets not in this situation are those only containing a self-packet, generated when no received in-packets are waiting to be forwarded. Therefore,  $M^* \leq M$ .

*Proposition 3* provides the expression of the mean of random variable  $M^*$  for uniform jitter, which comes immediately from the previous definition, the definition of mean and from (11).

**Proposition 3.** *In the conditions of Theorem 2, and with a uniformly distributed jitter within  $[0, J_m]$ , the mean of  $M(t)$  w.r.t. the jitter value is as follows:*

$$\begin{aligned} \mathbb{E}\{M^*\} &= \frac{\lambda_g}{\lambda_{in} + \lambda_g} + \frac{\lambda_{in}}{\lambda_{in} + \lambda_g} \mathbb{E}_t\{M\} = \\ &= \frac{\lambda_g}{\lambda_{in} + \lambda_g} + \frac{\lambda_{in}}{\lambda_{in} + \lambda_g} C_2 \left[ C_1 (\lambda_{in} + \lambda_g)^2 \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{J_m}{2\lambda_{in}}}\lambda_g}^{\sqrt{\frac{J_m}{2\lambda_{in}}}\lambda_{in} + \lambda_g} e^{-s^2} ds + \right. \\ &\quad \left. + 2\lambda_{in}^{\frac{3}{2}} + 2\lambda_g\lambda_{in}^{\frac{1}{2}} \left(1 - e^{-\frac{J_m}{2}(\lambda_{in} + 2\lambda_g)}\right) \right] \end{aligned} \quad (13)$$

Where  $C_1$  and  $C_2$  are defined as follows:

$$\begin{cases} C_1 &= \sqrt{2\pi J_m} e^{\frac{J_m}{2} \frac{(\lambda_{in} + \lambda_g)^2}{\lambda_{in}}} \\ C_2 &= \frac{1}{2\lambda_{in}^{\frac{3}{2}}} e^{-\frac{J_m}{2}(\lambda_{in} + 2\lambda_g)} \end{cases} \quad (14)$$

*Proof.* With probability  $\frac{\lambda_g}{\lambda_{in} + \lambda_g}$ , the first arrived packet after a transmission is a self-generated packet (which is transmitted immediately, and for which  $M^* = 1$ ). With probability  $1 - \frac{\lambda_g}{\lambda_{in} + \lambda_g} = \frac{\lambda_{in}}{\lambda_{in} + \lambda_g}$ , the first arrived packet after a transmission is an in-packet which triggers a collecting phase, and then  $M^* = M$ . Therefore,

$$\mathbb{E}\{M^*\} = \frac{\lambda_g}{\lambda_{in} + \lambda_g} + \frac{\lambda_{in}}{\lambda_{in} + \lambda_g} \mathbb{E}_t\{M\}$$

From the definition of the mean:

$$\mathbb{E}_t\{M(t)\} = \int_0^\infty M(t) f_{T_j}(t) dt = \frac{1}{J_m} \int_0^{J_m} (1 + N(t)) dt$$

Given that  $f_{T_j}(t) = \frac{1}{J_m}$  for  $t \in [0, J_m]$ , and  $f_{T_j}(t) = 0$  for  $t > J_m$  or  $t < 0$ . The result is immediate by using (11) and applying standard algebra.  $\square$

The result from *Proposition 3* allows to estimate the size increase of out-packets w.r.t. in-packets caused by jitter, as detailed in section 4.1 (eq. (6)), for the case that in-traffic dominates self-traffic ( $\lambda_{in} \gg \lambda_g$ ). Figure 5 shows the evolution of  $\mathbb{E}_t\{M\}$  depending on  $\lambda_{in}$ , for different values of  $\lambda_g$  and  $J_m = 1sec$ .

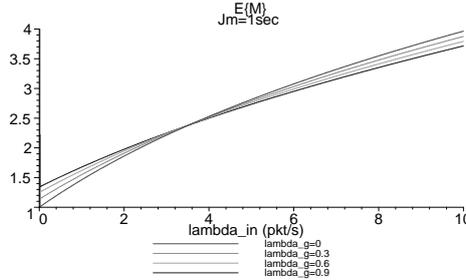


Figure 5: Mean of  $M$  depending on  $\lambda_{in}$ , for different values of  $\lambda_g$ ,  $J_m = 1sec$ .

Self-generated packets cause immediate transmission of out-packets. Therefore, increasing the self-packet rate reduces the duration of collecting phase in a greater extent than increasing the in-packet rate. It can be observed in Figure 5 that self-packet rate increase in  $\lambda_g$  causes a size growth of out-packets only for low values of  $\lambda_{in}$  ( $\lambda_{in} \leq 4 \frac{pkt}{sec}$  in the figure). For moderate and high values of  $\lambda_{in}$ , increases in self-packet rate lead to smaller out-packets. This is due to the fact that arriving self-generated packets are likely to cause an out-packet transmission before the arrival of in-packets that would have been otherwise included in the transmitted out-packet.

#### 4.4 Out-Packet Rate and Asymptotic Behavior

*Proposition 3* completes the characterization of the out-packet rate,  $\lambda_{out} = \frac{\lambda_{in} + \lambda_g}{\mathbb{E}\{M^*\}}$ , presented in (4). As  $M^* \geq 1$ , it can be expressed as  $M^* = N^* + 1$ , where  $N^* \geq 0$ . The equality holds only when the out-packet contains exactly one self-generated message, which happens with probability  $\frac{\lambda_g}{\lambda_{in} + \lambda_g}$ . In the rest of cases (*i.e.*, with probability  $\frac{\lambda_{in}}{\lambda_{in} + \lambda_g}$ ),  $N^*$  is equivalent to  $N$ , and therefore:

$$\lambda_{out} = \frac{\lambda_{in} + \lambda_g}{1 + \mathbb{E}\{N^*\}} = \frac{\lambda_{in} + \lambda_g}{1 + \frac{\lambda_{in}}{\lambda_{in} + \lambda_g} \mathbb{E}\{N\}} \quad (15)$$

As  $N = (\lambda_{in} + \lambda_g)D$ , (16) becomes:

$$\lambda_{out} = \frac{\lambda_{in} + \lambda_g}{1 + \lambda_{in} \mathbb{E}\{D\}} \quad (16)$$

Figure 6 illustrates the evolution of the out-packet rate,  $\lambda_{out}$ , with respect to the in- and self-packet rates ( $\lambda_{in}$  and  $\lambda_g$ ), for a constant value of  $J_m = 1sec$ . *Proposition 4* explores the asymptotic behavior of the out-packet rate in case of in-traffic and self-traffic dominance ( $\lambda_{in} \rightarrow 0$  and  $\lambda_g \rightarrow 0$ , respectively), as well as its compatibility with the no-jitter behavior ( $J_m \rightarrow 0$ ).

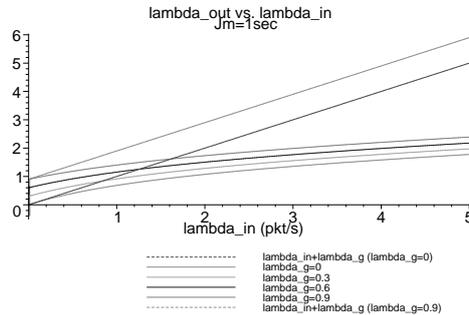


Figure 6:  $\lambda_{out}(\lambda_{in}, \lambda_g)$ , for  $J_m = 0.5sec$ .

**Proposition 4.** *The asymptotic behavior of the out-packet rate  $\lambda_{out}$  is as follows:*

$$\begin{aligned}\lim_{\lambda_g \rightarrow 0} \lambda_{out} &= \frac{\lambda_{in}}{1 + \lambda_{in} \mathbb{E}_t\{D(t)\}_{\lambda_g=0}} \\ \lim_{\lambda_{in} \rightarrow 0} \lambda_{out} &= \lambda_g \\ \lim_{J_m \rightarrow 0} \lambda_{out} &= \lambda_{in} + \lambda_g\end{aligned}$$

*Proof.* •  $\lambda_g \rightarrow 0$  and  $J_m \rightarrow 0$  are immediate from the definition.  
•  $\lambda_{in} \rightarrow 0$ :  $\lim_{\lambda_{in} \rightarrow 0} D(t)$  can be computed as follows.

$$\begin{aligned}D(t) &= \sqrt{\frac{\pi J_m}{2\lambda_{in}}} \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{J_m}{2\lambda_{in}}}\lambda_g}^{\sqrt{\frac{J_m}{2\lambda_{in}}}\left(\frac{\lambda_{in}t}{J_m} + \lambda_g\right)} e^{-s^2} ds \approx \\ &\approx \sqrt{\frac{\pi J_m}{2\lambda_{in}}} \frac{2}{\sqrt{\pi}} \left( e^{-\frac{J_m}{2\lambda_{in}}\left(\frac{\lambda_{in}t}{J_m} + \lambda_g\right)^2} - e^{-\frac{J_m}{2\lambda_{in}}\lambda_g^2} \right) \times \\ &\quad \times \frac{\lambda_{in}t}{J_m}\end{aligned}$$

Therefore:

$$\lim_{\lambda_{in} \rightarrow 0} D(t) = 0$$

Which does not depend on  $t$ ;  $\lim_{\lambda_{in} \rightarrow 0} \lambda_{out}$  can be therefore computed as the quotient of numerator and denominator limits, as follows:

$$\lim_{\lambda_{in} \rightarrow 0} \lambda_{out} = \lim_{\lambda_{in} \rightarrow 0} \frac{\lambda_{in} + \lambda_g}{1 + \lambda_{in} \mathbb{E}_t\{D\}} = \frac{0 + \lambda_g}{1 + 0 \times 0} = \lambda_g$$

□

The results of *Proposition 4* are consistent with the intuitive behavior of jitter. When jitter is not used, the out-packet rate corresponds to the addition of in- and self-packet rates. When self-traffic dominates over in-traffic ( $\lambda_g \gg \lambda_{in}$ , that is,  $\lambda_{in} \rightarrow 0$ ), the out-packet rate follows the self-packet rate, as self-generated packets cause immediate transmissions. In the inverse case, when self-traffic is negligible w.r.t. in-traffic ( $\lambda_g \ll \lambda_{in}$ , that is,  $\lambda_g \rightarrow 0$ ), the jitter mechanism reduces the packet rate in a ratio that corresponds to the number of in-packets received during a collecting phase ( $1 + \lambda_{in} \mathbb{E}_t\{D\}$ ).

## 4.5 Average Forwarding Delay for In-Packets

This section addresses the average delay that an in-packet experiences, given a jitter configuration (defined by uniform distribution of assigned jitter values within  $[0, J_m]$ ). Three steps are performed in order to characterize such delay. *Theorem 5* describes the cumulated delay of a collecting phase,  $G(t)$ , depending on the jitter value assigned to the triggering in-packet,  $t$ . The cumulated delay is the addition of the delays experienced by all in-packets (include the triggering

in-packet) that arrive within the collecting phase. The result is then particularized for the case of uniform jitter. Based on this result, *Theorem 6* computes the average delay for in-packets; and this is particularized in the *Corollary* for uniform jitter.

**Theorem 5.** *Let  $G(t)$  be the average cumulated delay in a collecting phase. Then, for an uniformly distributed jitter ( $T_j \sim \text{Uniform}[0, J_m]$ ), the expression of  $G(t)$  is as follows:*

$$G(t) = D(t) + F(t) \quad (17)$$

Where  $D(t)$  is defined in (10) and  $F(t)$  satisfies the following ODE:

$$F'(t) = \lambda_{in} D'(t) \int_0^t (1 - F_{T_j}(x)) dx \quad (18)$$

*Proof.* Let  $F(t)$  be the cumulated delay not corresponding to the one from the in-packet that triggers the collecting phase, *i.e.*  $F(t)$  is defined as  $F(t) = G(t) - D(t)$ . By restricting the time interval to a sufficiently small value of  $dt$  (as in proofs for *Theorems 1* and *2*), the arrival of a self-generated message implies that there is no additional delay. In case that an in-packet arrives, the additional delay corresponds to the total cumulated delay of such in-packet,  $G(x)$  if the assigned jitter  $x$  for such packet is smaller than  $t$ , and  $G(t)$  otherwise. The transition equation for  $F(t)$  is therefore as follows:

$$\begin{aligned} F(t + dt) &= \lambda_{in} dt (P(T_j > t)G(t) + \\ &\quad + \int_0^t P(T_j = x)G(x)dx) + \\ &\quad + \lambda_g dt 0 + (1 - (\lambda_{in} + \lambda_g)dt)F(t) \end{aligned}$$

$$\begin{aligned} \frac{F(t + dt) - F(t)}{dt} &= \lambda_{in} (P(T_j > t)G(t) + \\ &\quad + \int_0^t P(T_j = x)G(x)dx) - \\ &\quad - (\lambda_{in} + \lambda_g)F(t) \end{aligned}$$

When  $dt \rightarrow 0$ :

$$\begin{aligned} F'(t) &= \lambda_{in} (P(T_j > t)G(t) + \\ &\quad + \int_0^t P(T_j = x)G(x)dx) - \\ &\quad - (\lambda_{in} + \lambda_g)F(t) = \\ F'(t) &= \lambda_{in} (P(T_j > t)(G(t) - F(t)) + \\ &\quad + \int_0^t P(T_j = x)(G(x) - F(t))dx) - \\ &\quad - \lambda_g F(t) \end{aligned}$$

By definition,  $G(t) - F(t) = D(t)$ , therefore:

$$\begin{aligned} F'(t) &= \lambda_{in} (P(T_j > t)D(t) + \\ &\quad + \int_0^t P(T_j = x)(G(x) - F(t))dx) - \\ &\quad - \lambda_g F(t) \end{aligned}$$

Recalling that  $G(t) - F(t) = D(t)$ , and derivating again over  $t$ :

$$\begin{aligned} F''(t) &= \lambda_{in} (-f_{T_j}(t)D(t) + (1 - F_{T_j}(t))D'(t) + f_{T_j}(t)G(t) - \\ &\quad - F'(t)F_{T_j}(t) - F(t)f_{T_j}(t)) - \lambda_g F'(t) = \\ &= \lambda_{in}(1 - F_{T_j}(t))D'(t) - \lambda_{in}F_{T_j}(t)F'(t) - \lambda_g F'(t) = \\ &= \lambda_{in}(1 - F_{T_j}(t))D'(t) - (\lambda_{in}F_{T_j}(t) + \lambda_g)F'(t) \end{aligned}$$

Multiplying by  $D'(t)$  on both sides:

$$\begin{aligned} F''(t)D'(t) &= \lambda_{in}(1 - F_{T_j}(t))(D'(t))^2 - \\ &\quad - (\lambda_{in}F_{T_j}(t) + \lambda_g)F'(t)D'(t) \end{aligned}$$

From (9),  $D''(t) = -(\lambda_{in}F_{T_j}(t) + \lambda_g)D'(t)$  and therefore:

$$F''(t)D'(t) = \lambda_{in}(1 - F_{T_j}(t))(D'(t))^2 + D''(t)F'(t)$$

$$\begin{aligned} \frac{F''(t)D'(t) - D''(t)F'(t)}{D'(t)^2} &= \lambda_{in}(1 - F_{T_j}(t)) \\ \left( \frac{F'(t)}{D'(t)} \right)' &= \lambda_{in}(1 - F_{T_j}(t)) \end{aligned}$$

Applying the initial condition  $F'(0) = 0$ :

$$F'(t) = \lambda_{in} D'(t) \int_0^t (1 - F_{T_j}(x)) dx$$

□

**Corollary.** *If jitter values are distributed uniformly within  $[0, J_m]$ , according to (1), the differential equation (18) has the following solution for  $F(t)$ :*

$$\begin{aligned} F(t) &= e^{\frac{J_m}{2} \frac{\lambda_g}{\lambda_{in}}} \frac{J_m^{\frac{3}{2}}}{\sqrt{\lambda_{in}}} \sqrt{\frac{2}{\pi}} \int_{\sqrt{\frac{J_m}{2\lambda_{in}} \lambda_g}}^{\sqrt{\frac{J_m}{2\lambda_{in}} \lambda_g} + \sqrt{\frac{\lambda_{in}}{2J_m}} t} e^{-s^2} ds \times \\ &\quad \times \left( \lambda_g - \sqrt{\pi} \left( 2 + \frac{\lambda_g^2}{\lambda_{in}} \right) \right) + \frac{t}{2} e^{-\frac{\lambda_{in} t^2}{2J_m} - \lambda_g t} - \\ &\quad - \frac{J_m}{2} \left( 1 - e^{-\frac{\lambda_{in} t^2}{2J_m} - \lambda_g t} \right) \left( \frac{\lambda_g}{\lambda_{in}} + 2 \right) \end{aligned} \quad (19)$$

*Proof.* Applying the CDF of  $T_j$  in (18) leads to:

$$F'(t) = \lambda_{in} D'(t) \int_0^t \left(1 - \frac{x}{J_m}\right) dx = \lambda_{in} t \left(1 - \frac{t}{2J_m}\right) D'(t)$$

The result then comes from solving this equation with I.C.  $F(0) = 0$ .  $\square$

The average forwarding delay for an in-packet can be computed by using results obtained in previous sections. *Theorem 6* describes the average forwarding delay for in-packets received within a collecting phase with jitter value  $t$ .

**Theorem 6.** *The average delay between reception and retransmission for a message contained in an in-packet has the expression that follows:*

$$\overline{T_{tx}} = \int_0^{J_m} f_{T_j}(t) \frac{D(t) + F(t)}{1 + \lambda_{in} D(t)} dt \quad (20)$$

*Proof.* Within a collecting phase with jitter value  $t$ , the average cumulated delay of in-packets is  $G(t)$ . From (12), the average number of received in-packets is  $N_i(t) = \lambda_{in} D(t)$ . Therefore:

$$T_{tx}(t) = \frac{G(t)}{1 + N_i(t)} = \frac{D(t) + F(t)}{1 + \lambda_{in} D(t)}$$

Let  $T_{tx}$  be the random variable indicating the forwarding delay for an in-packet. Note that  $T_{tx} = T_{tx}(T_j)$  is a transform of the random variable  $T_j$  that indicates the jitter value assigned to the packet that triggers the collecting phase in which the considered in-packet arrives. Averaging over all possible jitter values for the collecting phase:

$$\overline{T_{tx}} = \mathbb{E}_t\{T_{tx}\} = \int_0^{J_m} f_{T_j}(t) \frac{D(t) + F(t)}{1 + \lambda_{in} D(t)} dt$$

$\square$

**Corollary.** *If jitter values are distributed uniformly, the average delay between reception and retransmission for a message contained in an in-packet is as follows:*

$$\overline{T_{tx}} = \frac{1}{J_m} \int_0^{J_m} \frac{D(t) + F(t)}{1 + \lambda_{in} D(t)} dt \quad (21)$$

## 5 Discussion

The above section analyzes the benefit of using of jitter in terms of packet transmission rate reduction. This has been modeled by studying the out-packet rate  $\lambda_{out}$  and its relationship with variables  $\lambda_{in}$  (in-packet rate) and  $\lambda_g$  (self-generated packet rate), for a uniformly distributed jitter selected within  $[0, J_m]$ . The expression of  $\lambda_{out} \equiv \lambda_{out}(\lambda_{in}, \lambda_g)|_{J_m}$  is detailed in equation (16). For the parameters chosen for representation in Figures 5 and 6 ( $\lambda_{in} = 4pkt/sec$ ,  $J_m = 1sec$ ) and  $\lambda_g = 0.2pkt/sec$ , for instance, the out-packet rate is  $\lambda_{out} =$

1.66pkt/sec, which implies a packet rate reduction (w.r.t. in-packet rate) of 60%. For a more realistic value of  $J_m = 100msec$ , the out-packet rate becomes  $\lambda_{out} = 3.49pkt/sec$ , still a significant reduction in the number of transmissions. Less transmissions are indeed very desirable in wireless ad hoc networks, where bandwidth scarcity and hard energy constraints are common.

However, this benefit comes at the cost of additional delay. This delay is accumulated while packets are buffered and waiting for the next backed-off transmission that is scheduled. When an in-packet is received by an interface using jitter, messages contained in the packet are forwarded after a random delay. If such in-packet triggers a new collecting phase, then the time lapse before forwarding corresponds to the length of the collecting phase, for which the average in function of the jitter value  $t$ ,  $D(t)$ , is described in equation (10). The length of the collecting phase is the upper bound or worst case for the random delay that an in-packet may experience. The average delay for in-packets, given a maximum jitter value  $J_m$ , is explored in equation (20). Even with a large jitter interval, as in the previous example ( $\lambda_{in} = 4pkt/sec$ ,  $\lambda_g = 0.2pkt/sec$ ,  $J_m = 1sec$ ), and from eq. (21), such average delay is  $T_{tx} = 0.24sec$ ; for the maximum jitter value  $J_m = 0.1sec$ ,  $T_{tx} = 0.04sec$ . These delays can be thus scaled into acceptable values with the jitter range. Based on the results presented in section 4, Figure 7 displays the plot of the average of the collecting phase length,  $D(T)$ , and the average delay for an in-packet,  $\mathbb{E}_t\{T_{tx}\}$  for an interface with in-packet traffic rate  $\lambda_{in} = 4\frac{pkts}{sec}$  and self-packet traffic rate  $\lambda_g = 0.2\frac{pkts}{sec}$ , where  $T$  is the average jitter value, distributed uniformly within  $[0, J_m]$  with  $J_m = 2T$ .

Moreover, the benefit of reduced out-packet transmission rate comes also at the cost of longer transmissions (out-packets), as shown in eqs. (5), (6) and (11). In practice, IETF standardization activity indicates that jitter is used at the network layer, generally above a link layer using CSMA based mechanisms (typically 802.11). The effect of longer packets with CSMA has been studied in various prior work including in [21], where it is shown that if  $L$  is the length of packets and  $B$  the bit error rate, the achieved throughput  $G$  is:

$$\frac{L}{L+1}(1-B)^{L+1} \quad (22)$$

Since the bit error rate is generally substantial in wireless ad hoc networks, this means that there is an optimum packet length, above which the throughput decreases. Therefore, the maximum jitter value  $J_m$  should be chosen so that packet size does not increase beyond the CSMA optimum, in which case the throughput would in fact decrease because of the link layer. In that respect, the choice of an appropriate  $J_m$  (with respect to out-packet size variation) also depends on  $\lambda_{in}$  and  $\lambda_g$ , as shown in *Proposition 3*.

## 6 Simulations

This section presents simulation results that focus on the two main results obtained in section 4: the delay introduced by jitter in packet forwarding (both the average length of the collecting phase and the average delay between in-packet reception and forwarding), and the relationship between out-packet traffic rate, self-packet traffic rate and in-packet traffic rate when jittering techniques are used. For a better visualization of the impact of the jitter range, each aspect

is measured for collecting phases with jitter values  $T = \frac{J_m}{2}$ , for different maximum values of the jitter,  $J_m$ . Results are presented for a range  $0 \leq T \leq 0.5 \text{sec}$ , or equivalently,  $0 \leq J_m \leq 1 \text{sec}$ , and directly compared with the corresponding analytical results that were derived in section 4. The simulations were carried out in Maple and the presented results are averaged over 30 iterations per value. 95% confidence intervals are indicated in the figures.

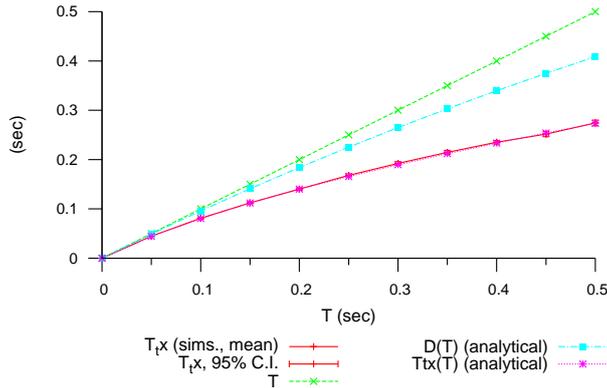


Figure 7: Average time to transmission for  $\lambda_{in} = 4 \frac{pkt}{s}$ ,  $\lambda_g = 0.2 \frac{pkt}{s}$ , for different values of  $t$  (simulations and analytical results).

Figure 7 presents the average delay for an in-packet,  $T_{tx}(T)$ , the average duration  $D(T)$  of the collecting phase (which corresponds to the delay for the triggering in-packet of the phase), together with the averaged results from the simulations. It can be observed that the obtained analytical results are consistent with the simulation results. This suggests that, with the simulated values, the transmission time of in-messages is frequently determined by the jitter values assigned to in-messages previously arrived, and the event that an in-packet arrival follows an out-packet transmission is rare. The probability of such event may increase when in-packet traffic rate decreases, thus approaching the measures of average in-packet delay,  $T_{tx}(T)$ , to the length of the collecting phase,  $D(T)$ .

Figure 8 displays the in-packet and out-packet rates obtained in simulations for different values of  $T$ , with a nominal in-packet rate of  $\lambda_{in} = 4 \frac{pkts}{sec}$  and self-packet rate of  $\lambda_g = 0.2 \frac{pkts}{sec}$ . Simulations are compared with the out-packet rate provided by the theoretical model via expression (16). It can be observed that the out-packet rate for  $T = 0$  corresponds to  $\lambda_{in} + \lambda_g = 4 + 0.2 \frac{pkts}{sec} = 4.2 \frac{pkts}{sec}$ . For non-zero average values of jitter, the out-packet rate decreases significantly as  $T$  grows. The slope of this decrease becomes lower (in absolute terms) as  $T$  value is higher. Although the range of simulated  $t$  is not long enough, the observed evolution is consistent with the horizontal asymptote at  $\lambda_{out} = \lambda_g = 0.2 \frac{pkts}{sec}$ , described in *Proposition 4*.

## 7 Conclusion

Recurrent network-wide packet dissemination may lead to systematic wireless collisions when performed over wireless multi-hop ad hoc networks. Jittering, a

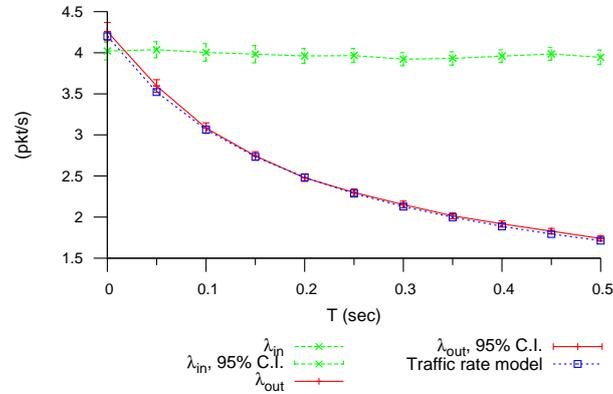


Figure 8: Out-packet ( $\lambda_{out}$ ) and in-packet ( $\lambda_{in}$ ) rates, for different values of  $T$ , and a theoretical in-packet rate  $\lambda_{in} = 4 \frac{pkt}{s}$  (simulations and analytical results).

distributed technique based on the schedule of random backed-off transmissions, aims at avoiding such transmissions bound to be synchronized otherwise. Jittering is moreover used to aggregate several packets (those received and buffered while waiting for the next backed-off transmission that has been scheduled) into a single transmission. Reducing the number of transmissions and the number of concurrent transmissions is very desirable in wireless ad hoc networks, where bandwidth is scarce, and energy supply often limited. However, the benefits of jitter come at the price of additional delays, and longer transmissions. This memorandum introduced a model and analysis of standard jittering as specified by the IETF, and derived results on three key aspects: (i) incurred additional delays, (ii) increase in packet size and (iii) reduction in the number of transmissions. This memorandum also presented an analysis of the use of jitter in practice, in conjunction with CSMA, the mechanism on which is based most current link layer technologies, such as 802.11. The analytical results are then validated via simulations. This memorandum thus provides a rather comprehensive analysis of the impact of standard jittering in today's wireless ad hoc networks. Future work will aim at extending the model to consider non-instantaneous packet transmissions, as well as a network-based approach (instead of the interface-based approach used so far) which may capture finer network-wide behavior.

## References

- [1] Cordero, J. A.; Jacquet, P.; Baccelli, E. (2011). RR 7701, *A Theoretical Model for Analyzing the Impact of Jitter-based Techniques on Flooding over Wireless Ad Hoc Networks*, INRIA Research Report, July 2011.
- [2] Roy, A. (ed.); Chandra, M. (2010). RFC 5820, *Extensions to OSPF to Support Mobile Ad Hoc Networking*, IETF, March 2010.
- [3] Friedman, R.; Hay, D.; Kliot, G. (2009). *Jittering Broadcast Transmissions in MANETs: Quantification and Implementation Strategies*.

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- [4] Baccelli, E.; Clausen, T.; Herberg, U.; Perkins, C. (2009). IP Links in Multihop Ad Hoc Wireless Networks?, *Proceedings of the 17th IEEE International Conference on Software Telecommunications and Computer Networks (SoftCOM'09)*, Split (Croatia), September 2009.
  - [5] Ogier, R.; Spagnolo, P. (2009). RFC 5614, *Mobile Ad Hoc Network (MANET) Extension of OSPF*, IETF, August 2009.
  - [6] Clausen, T. (2009). RR 6148, *MANET Router Configuration Recommendations*, INRIA Research Report, February 2009.
  - [7] Baccelli, E.; Jacquet, P.; Nguyen, D.; Clausen, T. (2009). RFC5449, *OSPF Multipoint Relay Extension for Ad Hoc Networks*, IETF, Feb.2009.
  - [8] Oliveira, R.;Bernardo, L.;Pinto, P. (2009).The influence of broadcast traffic on IEEE 802.11 DCF networks, In: *Computer Communications*, Vol. 32, No. 2, pp. 439-452, February 2009.
  - [9] Coltun, R.; Ferguson, D.; Moy, J.; Lindem, A. (ed.) (2008). RFC 5340, *OSPF for IPv6*, IETF, July 2008.
  - [10] Clausen, T.; Dearlove, C.; Adamson, B. (2008). RFC5148, *Jitter Considerations in Mobile Ad Hoc Networks*, IETF, Feb.2008.
  - [11] Chung, J.-M.; Min Soo, H.; Jeong, W.-C. (2004). Jitter Analysis of Homogeneous Traffic in Wireless Differentiated Services Networks, *Proceedings of the 13th IEEE Workshop on Local and Metropolitan Area Networks (LANMAN 2004)*, pp. 187-192, October 2004.
  - [12] Daniel, E. J.; White, C. M.; Teague, K. A. (2003). An Inter-arrival Delay Jitter Model using Multi-Structure Network Delay Characteristics for Packet Networks, *Proceedings of the 37th IEEE Asilomar Conference on Signals, Systems and Computers*, pp. 1738-1742, New York, NY (U.S.), November 2003.
  - [13] Perkins, C.; Belding-Royer, E.; Das, S. (2003). RFC 3561, *Ad hoc On-Demand Distance Vector (AODV) Routing*, IETF, July 2003.
  - [14] Sasson, Y.; Cavin, D.; Schiper, A. (2003). Probabilistic Broadcast for Flooding in Wireless Mobile Ad hoc Networks, *Proc. of the IEEE Intl. Conf. on Wireless Communications and Networking (WCNC2003)*, Vol. 2, pp. 1124-1130, New Orleans, LA (U.S.), May 2003.
  - [15] Clausen, T.; Jacquet, P. (2003). RFC 3626, *Optimized Link State Routing Protocol (OLSR)*, IETF, October 2003.
  - [16] Qayyum, A.; Viennot, L.; Laouti, A. (2002). Multipoint Relaying for Flooding Broadcast Messages in Mobile Wireless Networks, *Proceedings of the 35th Annual Hawaii International Conference on System Sciences (HICSS'02)*, Vol. 9, January 2002.
  - [17] Zheng, L.; Zhang, L.; Xu, D. (2001). Characteristics of network delay and delay jitter and its effect on voice over IP (VoIP), *Proceedings of the IEEE International Conference of Communications (ICC 2001)*, Vol. 1, pp. 122-126, Helsinki (Finland), June 2001.

- [18] Broch, J.; Maltz, D. A.; Johnson, D. B.; Hu, Y.-C.; Jetcheva, J. (1998). A performance comparison of multi-hop wireless ad hoc network routing protocols, *Proc. of ACM Intl. Conf. on Mobile Computing and Networking (MobiCom'98)*, pp.85-97, Dallas, TX (U.S.), Oct.1998.
- [19] Moy, J. (1998). RFC 2328, *OSPF Version 2*, IETF, April 1998.
- [20] Fulton, C. A.; Li, S. (1998). "Delay Jitter First-Order and Second-Order Statistical Functions of General Traffic on High-Speed Multimedia Networks", In: *IEEE/ACM Trans. on Networking*, Vol.6, No.2, Apr.1998.
- [21] Lettieri, P.; Srivastava, M. B. (1998). Adaptive Frame Length Control for Improving Wireless Link Throughput, Range, and Energy Efficiency. *Proceedings of INFOCOM'98*, Vol. 2, pp. 564-571, San Francisco, CA (U.S.), 1998.
- [22] Bisdikian, C. C.; Matragi, W.; Sohrawy, K. (1994). On the Jitter and Delay Analysis in ATM Multiplexer, *Proceedings of the IEEE International Conference on Communications (ICC'94)*, pp. 738-744, New Orleans, LA (United States), May 1994.
- [23] Takagi, H.; Kleinrock, L. (1984). Optimal Transmission Ranges for Randomly Distributed Packet Radio Terminals, In: *IEEE Transactions on Communications*, Vol. COM-32, No. 3, March 1984.
- [24] Dalal, Y. K.; Metcalfe, R. M. (1978). Reverse Path Forwarding of Broadcast Packets, In: *Communications of the ACM*, Vol. 21, Issue 12, December 1978.

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