An Overview of Analysis vs Synthesis in Low-Dimensional Signal Models

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Sparsity & Co.: An Overview of Analysis *vs* Synthesis in Low-Dimensional Signal Models

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***Abstract*—In the past decade there has been a great interest in a synthesis-based model for signals, based on sparse and redundant representations. Such a model assumes that the signal of interest can be composed as a linear combination of *few* columns from a given matrix (the dictionary). An alternative *analysis-based* model can be envisioned, where an analysis operator multiplies the signal, leading to a *cosparse* outcome. How similar are the two signal models ? The answer obviously depends on the dictionary/operator pair, and on the measure of (co)sparsity.**

For dictionaries in Hilbert spaces that are frames, the canonical dual is arguably the most natural associated analysis operator. When the frame is *localized*, the canonical frame coefficients provide a near sparsest expansion for several ℓ^p sparseness measures, $p \leq 1$. However, for frames which are not localized, this no longer holds true: the sparsest synthesis coefficients may differ significantly from the canonical coefficients.

In general the sparsest synthesis coefficients may also depend strongly on the choice of the sparseness measure, but this dependency vanishes for dictionaries with a *null space property* and signals that are combinations of sufficiently few columns from the dictionary. This uniqueness result, together with algorithmic guarantees, is at the basis of a number of signal reconstruction approaches for generic linear inverse problems (e.g., compressed sensing, inpainting, source separation, etc.).

Is there a similar uniqueness property when the data to be reconstructed is *cosparse* rather than sparse ? Can one derive cosparse regularization algorithms with performance guarantees ? Existing empirical evidence in the litterature suggests that a positive answer is likely. In recent work we propose a uniqueness result for the solution of linear inverse problems under a cosparse hypothesis, based on properties of the analysis operator and the measurement matrix. Unlike with the synthesis model, where recovery guarantees usually require the linear independence of sets of few columns from the dictionary, our results suggest that linear dependencies between rows of the analysis operators may be desirable.

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References

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