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Abstract. The article suggests a conceptual model-based simulation method with the aim to detect collision of cars in all-day road traffic. The benefit of the method within a driver assistance system would be twofold. Firstly, unavoidable accidents could be detected and appropriate actions like full braking maneuvers could be initiated in due course. Secondly, in case of an avoidable accident the algorithm is able to suggest an evasion trajectory that could be tracked by a future active steering driver assistance system. The algorithm exploits numerical optimal control techniques and reachable set analysis. A parametric sensitivity analysis is employed to investigate the influence of inaccurate sensor measurements.

Keywords: driver assistance, collision avoidance, optimal control, reachable sets, parametric sensitivity analysis

1 Introduction

Over the years many passive and active safety systems have been developed for modern passenger cars with the aim to reduce the number of casualties in traffic accidents. Passive safety systems contain amongst others improvements of the chassis, airbags, seat belts, and seat belt tighteners. These safety systems help to reduce the severeness of accidents once an accident has occured. In contrast, semi-active safety systems and driver assistance systems, for instance anti-blocking system, braking assistant, anti-slip regulation, electronic stability control, adaptive cruise control, lane departure warning, or blind spot intervention, become active in critical situations before an accident occurs and intend to prevent accidents. In future, active driver assistance systems that actively initiate braking maneuvers or even active steering maneuvers to avoid obstacles will become relevant in order to detect potential collisions and reduce severeness of collisions. The availability of high performance sensors will play a central role in future collision avoidance systems. But next to the required technical devices,

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intelligent software systems and algorithms will play a crucial role as well. The main tasks in collision avoidance are to reliably indicate future collisions and – if possible – to provide escape trajectories if such exist. This paper suggests an optimal control based method that has the potential of fulfilling these two tasks.

2 Model Scenarios

We investigate two model scenarios that are likely to occur in all-day traffic. According to [9] wrong velocity, short distance, and overtaking maneuvers are responsible for approximately 29.7 % of accidents with injuries to persons. The following model scenarios address these situations, see Figure 1. The typical time to collision ranges from 0.5 to 3 seconds. For simplicity we assume a straight road throughout. A reference coordinate system is used with the x-axis pointing into the longitudinal direction of the road and the y-axis pointing in the cross-direction of the road.

Scenario 1: A stationary obstacle at a given distance to an approaching car, which drives at a prescribed speed, has to be avoided.

Scenario 2: An overtaking maneuver on a highway is considered. One car (car A) has initiated an overtaking maneuver to overtake car B while another car (car C) is approaching at a prescribed speed.

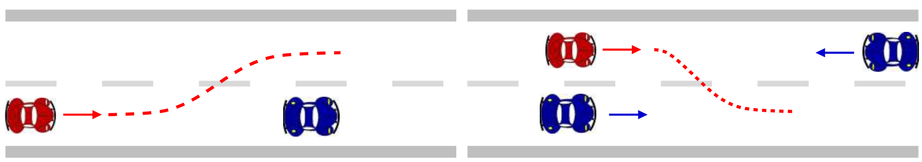

Fig. 1. Collision avoidance model scenarios: stationary obstacle (left) and overtaking maneuver (right)

For these two model scenarios we aim at answering the following questions:

- Can a collision be avoided?
- If a collision can be avoided, how can it be avoided?

3 Model of the Car

In this article the single-track car model is used. It is a simplified car model, which is commonly used in the automobile industry for basic investigations of the dynamical behavior of cars. It is based on the simplifying assumptions that rolling and pitching behavior of the car body can be neglected. The car model includes two control variables for the driver: the steering angle velocity $|w_\delta| \leq 0.5$ $[rad/s]$ and a function F_B with values in $[F_{Bmin}, F_{Bmax}]$, $F_{Bmin} = -5000$ $[N]$,

$F_{Bmax} = 15000\,[N]$, which models a combined brake (if $F_B > 0$) and acceleration (if $F_B < 0$) assembly. Details on the model can be found in [5–7]. The dynamics are given by the following system of differential equations for the car's center of gravity (x, y) in the plane, the yaw angle ψ, the velocities v_x and v_y in x- and y-direction, respectively, the yaw angle rate w_ψ, and the steering angle δ:

$$x' = v_x, \quad y' = v_y, \quad \psi' = w_\psi, \quad \delta' = w_\delta, \tag{1}$$

$$v_x' = \frac{1}{m}\left[F_x \cos\psi - F_y \sin\psi\right], \tag{2}$$

$$v_y' = \frac{1}{m}\left[F_x \sin\psi + F_y \cos\psi\right], \tag{3}$$

$$w_\psi' = \frac{1}{I_{zz}}\left[F_{sf} \cdot l_v \cdot \cos\delta - F_{sr} \cdot l_h + F_{lf} \cdot l_v \cdot \sin\delta\right], \tag{4}$$

The functions F_x, F_y, F_{sf}, F_{sr}, F_{lf} denote forces (in x-, y-direction, as well as lateral and longitudinal tyre forces at front and rear wheels) and are smooth nonlinear functions of the state $(x, y, \psi, v_x, v_y, w_\psi, \delta)$ and m, I_{zz}, l_v, l_h are constants. For further details please refer to [5–7]. For the following numerical computations we used realistic data for the various parameters involved in this model. Unfortunately, these parameter values are proprietary and may not be published. For a different parameter set which is quite realistic please refer to [5].

4 Collision Detection and Collision Avoidance

Once an obstacle has been detected by suitable sensors, e.g. radar or lidar, we use the following approaches to decide whether a collision is going to happen or not. As we intend to use optimal control to model the scenarios, as a by-product we obtain evasion trajectories if such exist at all. We investigate three different approaches. Herein, it is assumed for simplicity that the obstacle in scenario 1 is fixed close to the right boundary of a straight road as in the left picture in Figure 1. Moreover, the following approaches assume that the constellation of car and obstacle is such that a collision cannot be avoided by just applying a full braking maneuver.

4.1 Approach 1: Reaching a Safe Target Position for Scenario 1

The first approach aims at reaching a safe target state, which should be defined such that the evading car is able to avoid the obstacle and moreover is able to continue its drive after the obstacle has been passed. This approach is modeled by the following optimal control problem OCP$(y_0, v_{x,0})$:

Minimize

$$c_1 t_f + c_2 d + c_3 \int_0^{t_f} w_\delta(t)^2 dt$$

subject to the equations of motion (1)-(4) with initial condition

$$(x(0), y(0), \psi(0), v_x(0), v_y(0), w_\psi(0), \delta(0)) = (0, y_0, 0, v_{x,0}, 0, 0, 0),$$

the control constraints $|w_\delta| \le w_{\delta,max}$*,* $F_B \in [F_{B,min}, F_{B,max}]$*, the pure state constraint*

$$y_{min} \le y(t) \le y_{max},$$

and boundary conditions

$$x(t_f) = d, \quad v_y(t_f) = 0, \quad y(t_f) \ge y_{target}.$$

Herein, the final time t_f is supposed to be free and $c_1, c_2, c_3 \ge 0$ are suitable constants. y_0 is the initial y-position of the evading car on the road, $v_{x,0}$ is the initial velocity in x-direction of the evading car. y_{min} and y_{max} define the boundaries of the road. The terminal constraint $v_y(t_f) = 0$ shall ensure that the evading car can continue its drive beyond t_f without leaving the road immediately. y_{target} defines a y-position sufficiently far away from the obstacle's y-position (safe target position).

d is the initial distance of the evading car to the obstacle. If $c_2 > 0$, then d is assumed to be an additional optimization parameter. The resulting optimal control problem then aims at finding the minimal distance to the obstacle that still allows to avoid a collision. If $c_2 = 0$, then d is supposed to be a fixed distance. In this case, it might happen that the problem becomes infeasible owing to the constraint $y(t_f) \ge y_{target}$. A remedy in this case is approach 2 below.

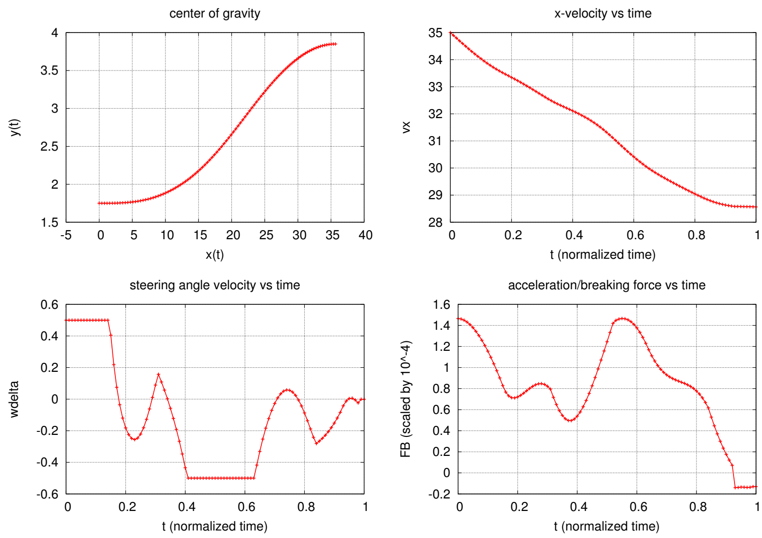

Fig. 2. Avoidance trajectory for $v_{x,0} = 35$, $y_0 = 1.75$, $y_{target} = 3.85$: center of gravity and velocity in x-direction (top), steering angle velocity and braking/acceleration force (bottom). The distance d computes to 35.6798 and the final time t_f to 1.1399.

Figure 2 shows the result for $y_{min} = 0.9$, $y_{max} = 6.1$, $y_0 = 1.75$, $v_{x,0} = 35$, $c_1 = c_2 = 0.1$, $c_2 = 0.2$, $w_{\delta,max} = 0.5$, $F_{B,min} = -5000$, $F_{B,max} = 15000$, $y_{target} = 3.85$. The minimal distance computes to $d = 35.6798$ [m] and the final time (time to collision) is $t_f = 1.1399$ [s].

Approach 1 yields one single optimal trajectory provided an avoidance trajectory exists. This avoidance trajectory could be tracked by a real car.

However, the solution depends on the definition of the safe target position y_{target} and it does not exist if a collision is unavoidable. Therefore it would be nicer to have full information about what points on the road can actually be reached by the evading car. This leads to the following reachable set approach.

4.2 Approach 2: Computing the Projected Reachable Set

The second approach aims at providing all points on the road that can be reached by the evading car in finite time t_f from a given initial state with boundary condition $v_y(t_f) = 0$. More precisely, we aim at computing the *projected reachable set*

$$\mathcal{PR} := \bigcup_{d\in[d_{min},d_{max}]} \bigcup_{y\in\mathcal{PR}(d)} \{(d,y)\},$$

where

$$\begin{aligned}\mathcal{PR}(d) := \big\{\hat{y} \in \mathbb{R} \mid \exists \text{ final time } t_f > 0,\ \text{controls } w_\delta, F_B,\\ &\text{and states } x, y, \psi, v_x, v_y, w_\delta, \delta \text{ such that}\\ &\text{dynamics and constraints are satisfied}\\ &\text{and } \hat{y} = y(t_f), x(t_f) = d, v_y(t_f) = 0\big\}\end{aligned}$$

denotes the projected reachable set at initial distance d. Note that we are not interested in the reachable set at t_f for the full state vector but only for the components x and y. In order to approximate the projected reachable set we employ the optimal control technique in [1, 2], which for a simplified setting allows a first order approximation. The set $\mathcal{PR}$ is approximated as follows. For $N, M \in \mathbb{N}$ and step-sizes $h = (d_{max} - d_{min})/N$ and $k = (y_{max} - y_{min})/M$ let

$$\mathbb{G}_{h,k} = \{(d_i, y_j) \in \mathbb{R}^2 \mid d_i = d_{min}+ih, y_j = y_{min}+jk, i = 0,\dots,N, j = 0,\dots,M\}$$

denote a grid covering the road region of interest. Then for each grid point $(d_i, y_j) \in \mathbb{G}_{h,k}$ the following optimal control problem is solved:

Minimize

$$\frac{1}{2}(y(t_f)) - y_j)^2$$

subject to the equations of motion (1)-(4) with initial condition

$$(x(0), y(0), \psi(0), v_x(0), v_y(0), w_\psi(0), \delta(0)) = (0, y_0, 0, v_{x,0}, 0, 0, 0),$$

the control constraints $|w_\delta| \le w_{\delta,max}$, $F_B \in [F_{B,min}, F_{B,max}]$, *the pure state constraint*

$$y_{min} \le y(t) \le y_{max},$$

and boundary conditions $x(t_f) = d_i$, $v_y(t_f) = 0$.

Let $x^*_{i,j}(\cdot)$ and $y^*_{i,j}(\cdot)$ denote the optimal solution components of the state vector. The projected reachable set is approximated by collecting all grid points in $\mathbb{G}_{h,k}$ with distance of order $\mathcal{O}(h+k)$ to end points of trajectories:

$$\mathcal{PR}_{h,k} := \bigcup_{\substack{(d_i,y_j)\in\mathbb{G}_{h,k}: \\ \|(x^*_{i,j}(t_f),y^*_{i,j}(t_f))-(d_i,y_j)\|\le C(h+k)}} \{(d_i,y_j)\}.$$

Herein, $C > 0$ is a constant. In [2] it is shown that the approximation $\mathcal{PR}_{h,k}$ convergences in the Hausdorff distance to $\mathcal{PR}$ of order $\mathcal{O}(h+k)$ as h and k approach zero, if $\mathcal{PR}$ is closed and non-empty. Direct discretization techniques for the numerical solution of the optimal control problem introduce a further approximation to $\mathcal{PR}_{h,k}$ whose convergence properties for a special setting are analyzed in [2] as well.

The projected reachable set approximations $\mathcal{PR}_{h,k}$ are depicted in Figure 3 for different initial velocities and the data $y_{min} = 1.3$, $y_{max} = 5.7$, $d_{min} = 10$, $d_{max} = 200$, $y_0 = 1.75$, $F_{B,min} = -5000$, $F_{B,max} = 15000$. The optimal control problems have been solved by the software OCPID-DAE1 [8].

Note that the obstacle car is not taken into account in the optimal control problems. But once the projected reachable set is known, it can be decided for a given obstacle position whether a collision can be avoided or not by investigating the remaining space in the projected reachable set outside the obstacle at the x-position of the obstacle.

4.3 Approach 3: Feasibility Problem for Scenario 2

For a fixed obstacle it is comparatively simple to define a safe target position or to approximate the projected reachable set, but for moving objects as in the overtaking maneuver in Figure 1 it is not, since a collision with all other moving cars has to be avoided at all times. In the overtaking scenario in Figure 1 let car A denote the car that overtakes a car called car B and car C is the car approaching car A in opposite direction. Cars B and C are supposed to drive at constant velocity in a straight line. Anti-collision constraints lead to the following pure state constraints, where W denotes the maximum width of the cars (for simplicity we use balls to model the anti-collision constraints):

$$(x_A(t) - x_B(t))^2 + (y_A(t) - y_B(t))^2 \ge W^2, \quad \text{(don't hit car B)}$$
$$(x_A(t) - x_C(t))^2 + (y_A(t) - y_C(t))^2 \ge W^2, \quad \text{(don't hit car C)}$$

Unfortunately, these constraints will be infeasible if there is no way to avoid a collision. Of course, in this case the resulting optimal control problems do not have a solution and numerical methods will fail. In order to circumvent this problem the relaxed constraints

$$(x_A(t) - x_B(t))^2 + (y_A(t) - y_B(t))^2 + \alpha \ge W^2,$$
$$(x_A(t) - x_C(t))^2 + (y_A(t) - y_C(t))^2 + \alpha \ge W^2$$

$v_{x,0} = 75$ km/h :

$v_{x,0} = 100$ km/h :

$v_{x,0} = 150$ km/h :

$v_{x,0} = 250$ km/h :

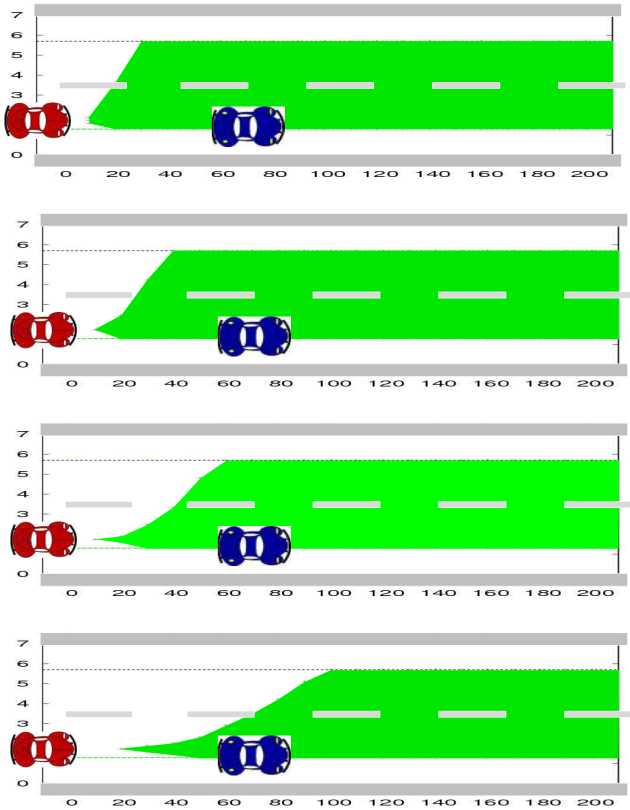

Fig. 3. Projected reachable sets for initial velocities $v_{x,0} = 75, 100, 150, 250$ [km/h]. The approaching car can avoid a collision with the obstacle in the first three settings, but not in the final setting, if the measurements of the obstacle and the approaching car are taken into account.

are considered, where α denotes the maximal constraint violation. Now, an optimal control problem with the aim to minimize the constraint violation α is solved subject to the above constraints. A collision detection algorithm is then given by considering the minimal constraint violation α^*. If $\alpha^* > 0$, then a collision cannot be avoided (the anti-collision constraints cannot be satisfied). If $\alpha^* \leq 0$, then a collision can be avoided with a trajectory that is produced by the optimal control problem. We illustrate the outcome for the following data:

- car A: 100 [km/h], car B: 75 [km/h], car C: 100 [km/h]
- car width 2.6 [m], road width 7 [m]
- initial y-position of car A : 5.25 [m]
  initial y-position of car B : 1.75 [m]
  initial y-position of car C : 5.25 [m]

Table 1 summarizes the results for different initial distances of cars A and C obtained with OCPID-DAE1 [8]. A movie that visualizes the overtaking maneuver with initial distance of 60 m can be downloaded on the homepage of the first author.

Table 1. Results for the constraint minimization problem for the overtaking maneuver.

initial distance [m]	constraint violation α^* [m]	collision
10	0.24780E+01	yes
20	0.22789E+01	yes
30	0.21355E+01	yes
40	0.19351E+01	yes
50	0.94517E-01	yes
60	0.74140E-08	no
$\vdots$	$\vdots$	$\vdots$
190	0.74593E-08	no
200	0.74760E-08	no

5 Sensor Influence

In the collision avoidance scenarios, the initial position, i.e the constellation of evading car and obstacles, is determined by sensor measurements. These sensor measurements are subject to measurement errors and hence it is important to investigate how the optimal solution depends on these sensor measurement errors. We outline this for approach 1 and consider the initial values y_0 and $v_{x,0}$ to be parameters in the optimal control problem $\mathrm{OCP}(y_0, v_{x,0})$. We apply the sensitivity analysis in [3], which exploits the sensitivity results in [4] for finite dimensional optimization problems. To this end, $\mathrm{OCP}(y_0, v_{x,0})$ is discretized using piecewise constant control approximations $w_{\delta,j} \approx w_\delta(t_j)$ and $F_{B,j} \approx F_B(t_j)$, $j = 0, \ldots, K$, and Runge-Kutta approximations for the state on a grid with grid points t_j, $j = 0, \ldots, K$. The discretized problem is a finite dimensional nonlinear optimization problem, which is solved for nominal parameters $\hat{y}_0$ and $\hat{v}_{x,0}$. If the nominal optimal solution satisfies the assumptions of the sensitivity theorem in [4], i.e. second order sufficient conditions, linear independence constraint qualification and strict complementarity, then it was shown that the solution locally depends continuously differentiable on the parameters y_0 and $v_{x,0}$ and the sensitivities of the optimal control discretization with respect to the initial values y_0 and $v_{x,0}$ can be computed, that is we obtain the sensitivities

$$\frac{dw_{\delta,j}}{(y_0, v_{x,0})}(\hat{y}_0, \hat{v}_{x,0}), \quad \frac{dF_{B,j}}{(y_0, v_{x,0})}(\hat{y}_0, \hat{v}_{x,0}), \quad j = 0, \ldots, K.$$

The sensitivities indicate how sensitive the solution depends on perturbations in the initial values and hence can help to specify tolerances for sensors. We omit the details here since this sensitivity approach became quite standard in the meanwhile. Details can be found, e.g., in [3].

Figure 4 shows the sensitivity differentials for the controls w_δ and F_B with respect to y_0 and $v_{x,0}$ at the nominal parameters $\hat{y}_0 = 1.75$ and $\hat{v}_{x,0} = 35$. From the pictures it can be concluded that a perturbation in the initial y-position has the highest influence on the controls w_δ and F_B. A perturbation of the initial velocity $v_{x,0}$ has less influence. Hence, the sensor measurement of the y-position (respectively, the offset to the obstacle) should be more accurate than the sensor measurement of the velocity. More elaborate investigations regarding the definition of sensor tolerances that are necessary to achieve a certain performance are currently under investigation.

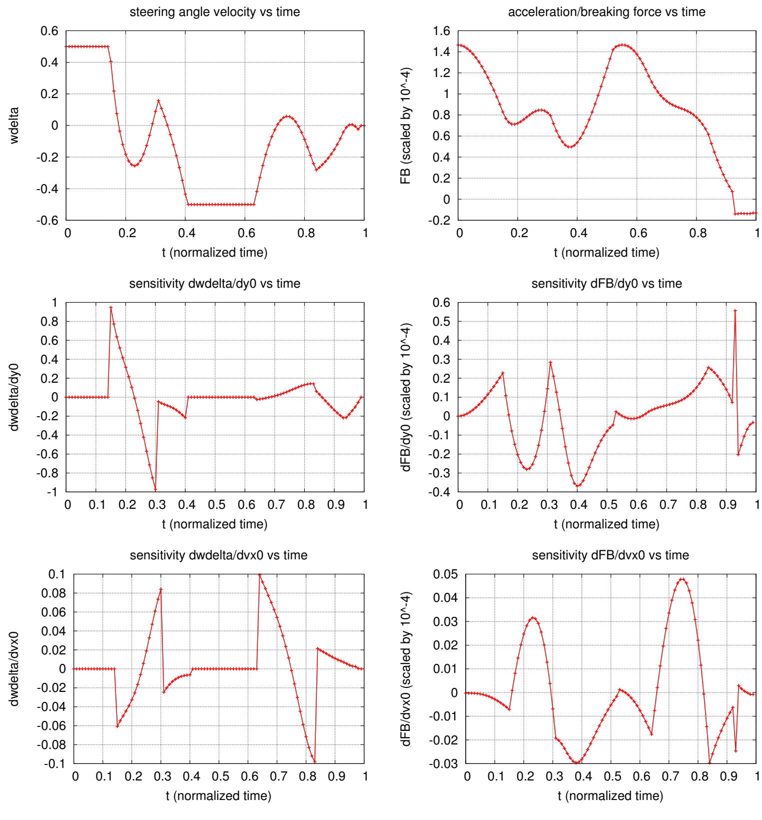

Fig. 4. Sensitivity differentials for the optimal controls of OCP$(\hat{y}_0, \hat{v}_{x,0})$ at $\hat{y}_0 = 1.75$ and $\hat{v}_{x,0} = 35$: Control and sensitivities of w_δ w.r.t. y_0 and $v_{x,0}$ (left) and of F_B w.r.t. y_0 and $v_{x,0}$ (right).

Please note that the above sensitivity approach does not work for the optimal control problems in approach 2 as those do not satisfy the assumptions of the sensitivity theorem in [4] whenever a grid point is in the projected reachable set. Adding a regularization term in the objective function might help to overcome this difficulty and would allow to investigate the dependence of the projected reachable set on sensor measurements.

6 Outlook

The paper suggests different approaches to an avoidance trajectory system based on optimal control techniques, reachable set computations, and sensitivity analysis. Many extensions are possible, e.g. computation of driver-friendly trajectories for active steering driver assistance systems, more complicated road geometries, real-time approximations, investigation of worst-case scenarios or cooperative control in the presence of many moving objects, and the investigation of parameter dependence of the projected reachable set. These issues are currently under investigation.

References

1. Baier, R., Gerdts, M.: A computational method for non-convex reachable sets using optimal control. In Proceedings of the European Control Conference (ECC) 2009, Budapest (Hungary), August 23-26, pp. 97–102, (2009)
2. Baier, R., Gerdts, M., Xausa, I.: Approximation of Reachable Sets using Optimal Control Algorithms, Technical report, submitted (2011)
3. Büskens, C., Maurer, H.: Sensitivity Analysis and Real-Time Control of Parametric Optimal Control Problems Using Nonlinear Programming Methods. In Grötschel, M., Krumke, S.O., Rambau, J. (eds): Online Optimization of Large Scale Systems, pp. 56–68. Springer (2001)
4. A. V. Fiacco, A.V.: Introduction to Sensitivity and Stability Analysis in Nonlinear Programming, Mathematics in Science and Engineering 165, Academic Press, New York (1983)
5. Gerdts, M.: Solving Mixed-Integer Optimal Control Problems by Branch&Bound: A Case Study from Automobile Test-Driving with Gear Shift. Optimal Control, Applications and Methods 26(1), 1–18 (2005)
6. Gerdts, M.: A variable time transformation method for mixed-integer optimal control problems. Optimal Control, Applications and Methods 27(3), 169–182 (2006)
7. Gerdts, M., Karrenberg, S., Müller-Beßler, B., Stock, G.: Generating Optimal Trajectories for an Automatically Driven Car, Optimization and Engineering 10 (4), pp. 439–463 (2009)
8. Gerdts, M.: OCPID-DAE1 – Optimal Control and Parameter Identification with Differential-Algebraic Equations of Index 1: User's Guide. Technical report, Institut für Mathematik und Rechneranwendung, Universität der Bundeswehr München (2011)
9. Statistisches Bundesamt: Unfallentwicklung auf deutschen Strassen 2008, Statistisches Bundesamt, Wiesbaden, `http://www.destatis.de` (2009).